Quantum Information: Homework 4

Due: 17 September 2018

1 Photon polarization states

Describe the following polarization states of a single photon in terms of $\{|{\rightarrow}\rangle, |{\uparrow}\rangle\}$.

a) A linearly polarized photon with polarization axis along $(\hat{\mathbf{x}} + \hat{\mathbf{y}})/\sqrt{2}$.

b) A linearly polarized photon with polarization axis along $(\hat{\mathbf{x}} - \hat{\mathbf{y}})/\sqrt{2}$.

c) A linearly polarized photon with polarization axis at angle $\pi/6$ above the x axis.

d) An elliptically polarized photon whose (classical) electric field traces an ellipse counterclockwise and whose major axis is along y and twice as large as that along x.

2 Polarization measurements

a) The following questions concern linearly polarized states along the axis at an angle θ counterclockwise from the x axis. Suppose that the classical wave is described with an electric field with amplitude E_0. Determine expressions for E_{ox} and E_{oy} for this. Use these to construct the associated single photon state in terms of $\{|{\rightarrow}\rangle, |{\uparrow}\rangle\}$. Construct the orthogonal polarization state.

b) Suppose that a linearly polarized photon has polarization axis at angle θ counterclockwise from the x axis. This is subjected to a PBS that measures horizontal versus vertical polarization in the configuration described in class. Determine the probabilities with which each measurement outcome occurs.

c) Suppose that a left circularly polarized photon is subjected to a measurement that measures linear polarization along the axis system rotated at an angle θ counterclockwise from the x axis. Determine the probabilities of the two measurement outcomes.

d) Suppose that a right circularly polarized photon is subjected to a measurement that measures linear polarization along the axis system rotated at an angle θ counterclockwise from the x axis. Determine the probabilities of the two measurement outcomes.

e) Can a linear polarization measurement distinguish between a collection of photons in the $|L\rangle$ state versus a collection in the $|L\rangle$ state? Explain your answer.

3 Basis change

The generic state of a qubit is

$$|\psi\rangle = c_0 |0\rangle + c_1 |1\rangle .$$

a) Express this in terms of the basis

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|\phi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle .$$

b) Express this in terms of the basis

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$
$$|\phi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle .$$

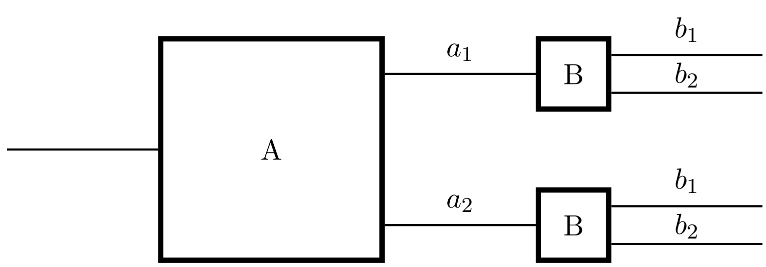
4 Qubit measurements

Various people propose to build measuring devices for single qubits and they list the states associated with the measurement outcomes. For each of teh following pairs of states, explain whether the measurement is feasible

a) $(|0\rangle + |1\rangle)/\sqrt{2}$ and $(|0\rangle + i|1\rangle)/\sqrt{2}$
b) $(|0\rangle - |1\rangle)/\sqrt{2}$ and $(|0\rangle + i|1\rangle)/\sqrt{2}$
c) $(3|0\rangle + 4|1\rangle)/5$ and $(4|0\rangle + 3i|1\rangle)/5$
d) $(3|0\rangle + 4i|1\rangle)/5$ and $(4|0\rangle - 3i|1\rangle)/5$
e) $(3|0\rangle + 4i|1\rangle)/5$ and $(|0\rangle + i|1\rangle)/\sqrt{2}$

5 Generic qubit measurements

Each of the following considers a sequence of cascaded measurements as illustrated.



In each case determine the probabilities of all four pairs of outcomes.

a) A measures in the basis $\{|0\rangle, |1\rangle\}$ and B in the basis

$$\left\{\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle, \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right\}$$

The state prior to A is $|0\rangle$.

b) A measures in the basis $\{|0\rangle, |1\rangle\}$ and B in the basis

$$\left\{\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle, \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right\}$$

The state prior to A is $\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$.

c) A measures in the basis $\{|0\rangle, |1\rangle\}$ and B in the basis

$$\left\{\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle, \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right\}$$

The state prior to A is $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$.

d) A measures in the basis

$$\left\{\frac{1}{\sqrt{5}}|0\rangle + \frac{2i}{\sqrt{5}}|1\rangle, \frac{2i}{\sqrt{5}}|0\rangle + \frac{1}{\sqrt{5}}|1\rangle\right\}$$

and B in the basis

$$\left\{\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle, \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle\right\}$$

The state prior to A is $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$.

6 Reiffel, *Quantum Computing*, 2.14, page 30.

7 News item

Find a news item published within the last year that describes an advance in quantum computing or quantum information. Post a link to the item in the discussion thread for 18 September. Summarize (here) in a single paragraph what the article describes.