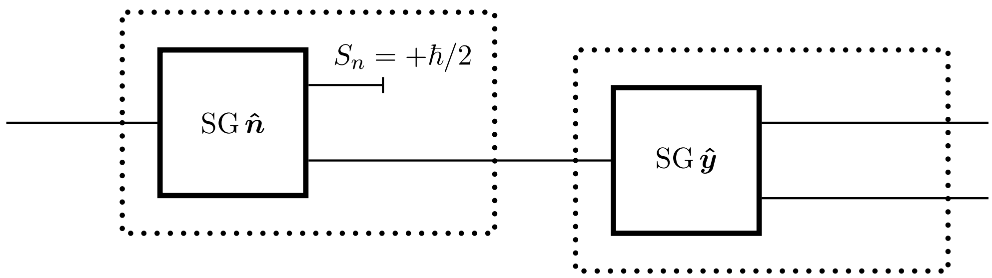
Quantum Information: Homework 2

Due: 4 September 2018

1 Repeated Stern-Gerlach experiments

Two Stern-Gerlach devices are arranged as illustrated. The first is oriented in the direction $\hat{\mathbf{n}} = (\hat{\mathbf{x}} + 2\hat{\mathbf{y}})/\sqrt{5}$ and is such that the beam corresponding to $S_n = +\hbar/2$ is blocked. The second is oriented along the direction $\hat{\mathbf{y}}$. Neither of its outgoing beams are blocked.



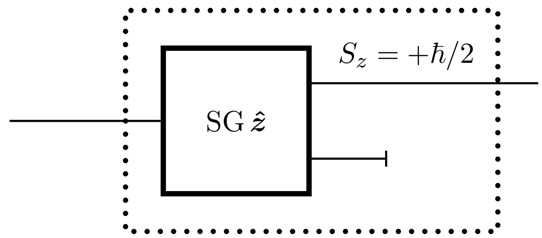
a) Suppose that a spin-1/2 particle initially in the state $|+\hat{\boldsymbol{z}}\rangle$ is subjected to the first device. Determine the probability with which it emerges from the leftmost dotted box.

b) Consider a particle that emerges from the first device and which is then subjected to the second Stern-Gerlach apparatus. List the possible outcomes of the second measurement, S_y, and the probabilities with which they will occur (for a particle which emerges from the first device with 100% certainty).

c) Suppose that a spin-1/2 particle is initially in the state $|+\hat{\boldsymbol{z}}\rangle$ prior to the first measurement. Determine the probability with which it yields $S_y = +\hbar/2$ after the second device. Determine the probability with which it yields $S_y = -\hbar/2$ after the second device. Do these probabilities add to 1? If not, why not?

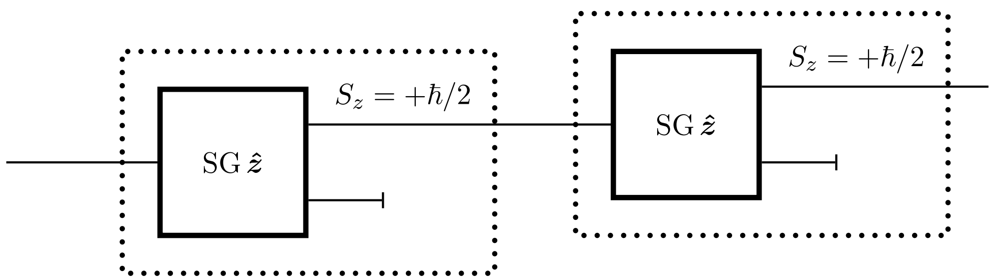
2 Repeatable vs non-repeatable measurements.

Consider a measuring device, which contains an SG $\hat{\boldsymbol{z}}$ apparatus combined with a blocking device on the lower outgoing beam. This is packaged within the dotted box as illustrated. Suppose that *a single* particle is sent into the boxed device. The idea will be to subject the *same* particle to repeated measurements.



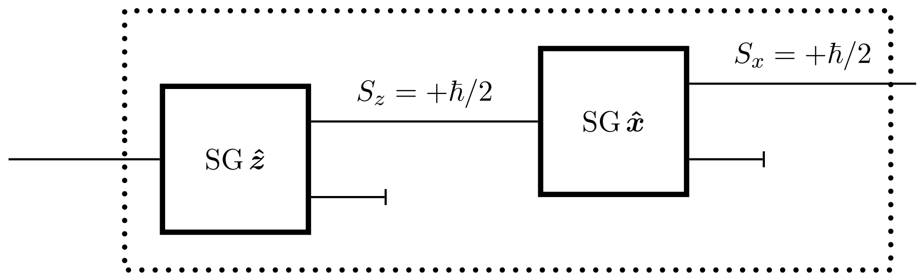
This apparatus is a measuring device in the sense that it answers the question: "Is $S_z = +\hbar/2$ for the particle entering the apparatus?" If particle emerges from the dotted box then a reasonable answer could be "yes." However, this is only reasonable because the measurement can be repeated and always yields the same outcomes in the sense described below.

Suppose that a particle which emerges from the dotted box is immediately subjected to the same device again. (Note: it is important to realize that the *same* particle is subjected to both devices. The situation is very different if one particle is subject to the first device and *another* particle subject to the second device.)



Any particle which emerges from the first boxed apparatus (i.e. answers the question with "yes") will emerge with 100% certainty from the boxed second apparatus (i.e. will answer the question when repeated with "yes" with 100% certainty). One can repeatedly add other boxed apparati like this and any particle which emerges from the first necessarily emerges from all the others. This is equivalent to repeatedly asking "Is $S_z = +\hbar/2$ for the particle entering the apparatus?" and always receiving the answer "yes." In this sense it is reasonable to say that $S_z = +\hbar/2$ for this particle. This is an example of a *repeatable measurement.*

Now consider the composite measurement, involving two blocked SG measuring devices with different orientations as illustrated.



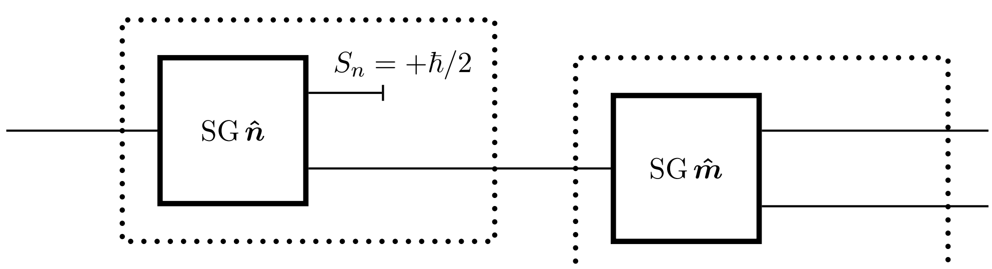
This composite SG apparatus (inside the dotted box) may appear as a single measuring device which asks the question "Is $S_z = +\hbar/2$ **and** $S_x = +\hbar/2$ for the particle entering the apparatus?" and it may seem that the answer should be "yes" for a particle which emerges from the combined apparatus.

a) This will only make sense if the measurement is repeatable. That is, one must take a particle which emerges from the dotted box and feed it back into a second copy of the entire dotted box. It must emerge with 100% certainty after this repetition for the measurement to be repeatable. Determine the probability with which a particle that emerges from the dotted box will do so again if it is fed into another copy of the dotted box. Is this measurement repeatable? Does it make sense to say that $S_z = +\hbar/2$ **and** $S_x = +\hbar/2$ for the particle?

b) Based on this result, why do you suspect that we have not tried to describe the state of a spin-1/2 particle in terms of more than one spin component?

3 Repeated Stern-Gerlach experiments

Two Stern-Gerlach devices are arranged as illustrated. The first is oriented in the direction $\hat{\mathbf{n}} = (4\hat{\mathbf{x}} + 3\hat{\mathbf{y}})/5$ and is such that the beam corresponding to $S_n = +\hbar/2$ is blocked. The second is oriented along the direction $\hat{\mathbf{m}}$. Neither of its outgoing beams are blocked.



a) Explain the choice of direction $\hat{\mathbf{m}}$ such that no particles emerge from the second Stern-Gerlach apparatus.

b) Explain the choice of direction $\hat{\mathbf{m}}$ such that particle are equally likely to emerge from either output of the second Stern-Gerlach apparatus.

4 Ket operations

Consider the kets

$$\begin{aligned} |\phi\rangle &= 5\,|+\hat{\mathbf{z}}\rangle + 12\,|-\hat{\mathbf{z}}\rangle\,, \\ |\psi\rangle &= 3i\,|+\hat{\mathbf{z}}\rangle - 4i\,|-\hat{\mathbf{z}}\rangle\,, \text{and} \\ |\chi\rangle &= 1\,|+\hat{\mathbf{z}}\rangle + 3i\,|-\hat{\mathbf{z}}\rangle\,. \end{aligned}$$

a) Determine whether each ket is normalized. If not, describe how to normalize it, e.g. find A so that $A\,|\phi\rangle$ is normalized.

b) Determine the inner products of all pairs of these kets.

c) For each of these kets, find another that is orthogonal to it.

5 More ket operations

Consider the kets

$$\begin{aligned} |\phi\rangle &= \frac{1}{\sqrt{13}}\left(2\,|+\hat{\mathbf{z}}\rangle + 3\,|-\hat{\mathbf{z}}\rangle\right) \text{ and} \\ |\psi\rangle &= \frac{1}{\sqrt{13}}\left(2\,|+\hat{\mathbf{z}}\rangle + i3\,|-\hat{\mathbf{z}}\rangle\right) \end{aligned}$$

Let

$$|\chi\rangle := \frac{1}{\sqrt{2}}\,|\phi\rangle + \frac{1}{\sqrt{2}}\,|\psi\rangle\,.$$

a) Show that each of $|\phi\rangle$ and $|\psi\rangle$ is normalized.

b) Express $|\chi\rangle$ as a combination of $|+\hat{\mathbf{z}}\rangle$ and $|-\hat{\mathbf{z}}\rangle$.

c) Check whether $|\chi\rangle$ is normalized.

6 Basis kets

Consider the pair of kets

$$\begin{aligned} |\phi_1\rangle &= \frac{1}{\sqrt{2}}\,|+\hat{\mathbf{z}}\rangle + \frac{1}{\sqrt{2}}\,|-\hat{\mathbf{z}}\rangle\,, \\ |\phi_2\rangle &= \frac{1}{\sqrt{2}}\,|+\hat{\mathbf{z}}\rangle - \frac{1}{\sqrt{2}}\,|-\hat{\mathbf{z}}\rangle \end{aligned}$$

and also the pair of kets

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{\sqrt{2}}\,|+\hat{\mathbf{z}}\rangle + \frac{i}{\sqrt{2}}\,|-\hat{\mathbf{z}}\rangle\,, \\ |\psi_2\rangle &= \frac{1}{\sqrt{2}}\,|+\hat{\mathbf{z}}\rangle - \frac{i}{\sqrt{2}}\,|-\hat{\mathbf{z}}\rangle\,. \end{aligned}$$

a) Express $|+\hat{\mathbf{z}}\rangle$ and $|-\hat{\mathbf{z}}\rangle$ as linear combinations of $|\phi_1\rangle$ and $|\phi_2\rangle$. That is write each in the form:

$$|+\hat{\mathbf{z}}\rangle = \text{number}\,|\phi_1\rangle + \text{number}\,|\phi_2\rangle$$

b) Express $|+\hat{\mathbf{z}}\rangle$ and $|-\hat{\mathbf{z}}\rangle$ as linear combinations of $|\psi_1\rangle$ and $|\psi_2\rangle$.

c) Express $|\psi_1\rangle$ and $|\psi_2\rangle$ as linear combinations of $|\phi_1\rangle$ and $|\phi_2\rangle$.

d) Show that each of $|\phi_1\rangle$ and $|\phi_2\rangle$ is normalized and show that these are orthogonal.

e) Show that each of $|\psi_1\rangle$ and $|\psi_2\rangle$ is normalized and show that these are orthogonal.

f) Determine $\langle\phi_i|\psi_j\rangle$ and $|\langle\phi_i|\psi_j\rangle|^2$ for all combinations of i and j.

g) Describe whether it is possible that $|\phi_1\rangle$ and $|\phi_2\rangle$ are associated with the two outcomes of *one* particular measurement.

h) Describe whether it is possible that $|\psi_1\rangle$ and $|\psi_2\rangle$ are associated with the two outcomes of *one* particular measurement.

i) Describe whether it is possible that $|\phi_1\rangle$ and $|\psi_2\rangle$ are associated with the two outcomes of *one* particular measurement.

j) Describe whether it is possible that $|\psi_1\rangle$ and $|\phi_2\rangle$ are associated with the two outcomes of *one* particular measurement.

7 News item

Find a news item published within the last year that describes an advance in quantum computing or quantum information. Post a link to the item in the discussion thread for 4 September. Summarize (here) in a single paragraph what the article describes.