Phys 396
Fall 2018

Quantum Information: Homework 1

Due: 28 August 2018

1 Binary addition

For each of the following pairs of numbers, represent each number in binary form, add the numbers using binary form and convert the result to a decimal number. Verify that the procedure was correct.

a) 9 and 11.

b) 9 and 15.

c) 35 and 124.

d) 76 and 85.

2 Binary multiplication

For each of the following pairs of numbers, represent each number in binary form, multiply the numbers using binary form and convert the result to a decimal number. Verify that the procedure was correct.

a) 5 and 6.

b) 4 and 9.

c) 18 and 25.

3 Modular addition

The following problems involve identities with modular addition. To prove them, consider a single binary digit, a, and prove these by checking that they are true for all possible choices of a.

a) Show that $a \oplus a = 0$.

b) Show that $a \oplus 1 = \overline{a}$.

c) Show that $a \oplus \overline{a} = 1$.

4 Binary addition

Consider two numbers in binary form

$$a = a_{n-1}a_{n-2}\dots a_1 a_0$$
$$b = b_{n-1}b_{n-2}\dots b_1 b_0$$

Let $s = a + b$. This will have form

$$s = s_n s_{n-2} \dots s_1 s_0$$

and the question is how to determine the binary digits of s from those of a and b. Clearly the digit s_k will depend on a_k and b_k. But it will also depend on the "carry," c_k that from the addition on the $k-1$ bit.

a) Show, by checking all possible bit values, that $s_k = a_k \oplus b_k \oplus c_k$.
b) Show, by checking all possible bit values, the carry from the addition of the k bits is $c_{k+1} = a_k b_k \oplus c_k a_k \oplus c_k b_k$.
c) Check that this scheme works to correctly add $011 + 001$. Note that the carry on the bit zero is automatically $c_0 = 0$.
d) Suppose that two n bit numbers need to be added. Counting each modular addition of two single digits as one and each multiplication of two single digits as one, determine the total number of such operations needed to add two n bit numbers.

5 Logic operations

In the following a and b represent single binary digits.

a) Show that
$$a \vee b = \overline{\overline{a} \wedge \overline{b}}.$$
b) Show that
$$a \wedge b = \overline{\overline{a} \vee \overline{b}}.$$

6 XOR gate

In the following a and b represent single binary digits.

a) Show that the binary sum of these, $s = a \oplus b$ satisfies
$$s = (\overline{a} \wedge b) \vee \left(a \wedge \overline{b}\right).$$
b) Use the previous result to show how to construct an XOR gate from AND, OR and NOT gates.

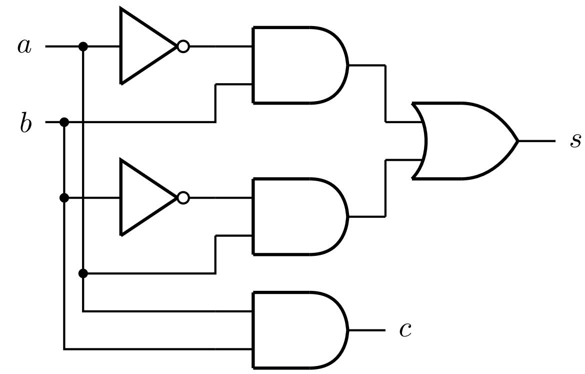
7 Basic gates

DeMorgan's laws will be useful in the following.

a) Provide a circuit that constructs an OR gate entirely from NAND gates.
b) Provide a circuit that constructs an XOR gate entirely from NAND gates.

8 Half-adder

Suppose that a and b are two binary digits. Consider the illustrated half-adder circuit.



a) Show that this produces the sum $s = a \oplus b$.

b) Show that this produces the carry $c = ab$.