# Laboratory 7: Charging and Discharging a Capacitor – Prelab

Consider a capacitor with capacitance $C$ connected in series to a resistor with resistance $R$ as shown in Fig. 1. Theory predicts that the potential difference across the capacitor is

$$V_C = V_{C0}\, e^{-t/\tau}$$

where $\tau = RC$ is the **capacitive time constant** and $V_{C0}$ is the the potential difference at time $t = 0$.

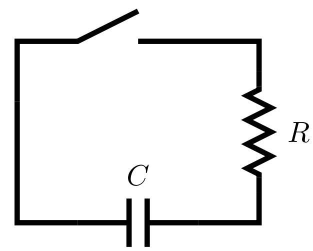


Figure 1: Capacitor and resistor in series.

## 1 Decay rate

In a particular situation, $C = 500\,\mu\text{F}$ and $R = 2200\,\Omega$ and at $t = 0$ the potential across the capacitor is $5.0\,\text{V}$. Determine the amount of time that must pass until the potential across the capacitor reaches half its initial value.

## 2 Linearized graph

Again suppose that $C = 500\,\mu\text{F}$ and $R = 2200\,\Omega$ and at $t = 0$ the potential across the capacitor is $5.0\,\text{V}$. If a graph of $\ln(V_C)$ versus $t$ were plotted it should yield a straight line. Determine the slope of this line.

# Laboratory 7: Charging and Discharging a Capacitor – Activity

The currents and potential differences in series circuits which contain capacitors constantly change as the capacitor charges and discharges. In this lab you will develop and explore a detailed description of a discharging capacitor; additionally this will provide method for measuring capacitance accurately.

The fundamental relationship which dictates the capacitor's behavior is that the excess positive charge on one plate of the capacitor is

$$q = C\Delta V \tag{1}$$

where $\Delta V$ is the potential difference across the capacitor plates and $C$ is the capacitance. The current that flows into or out of the capacitor is

$$I = \frac{dq}{dt}. \tag{2}$$

Most of this experiment considers a capacitor connected in series to a resistor as shown in Fig. 2. We aim to able to predict the following as functions of time $t$: the excess positive charge on one plate of the capacitor $q$, the current through the resistor $I$, and the voltage across the capacitor by $V_C$. Denote the resistance of the resistor by $R$ and the capacitance of the capacitor by $C$. The direction of the current relative to the charge on the capacitor plates is illustrated in Fig. 2. It follows that

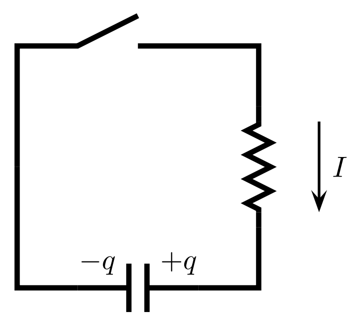


Figure 2: Capacitor and resistor in series: discharging mode.

$$I = \frac{dq}{dt} \tag{3}$$

whether the capacitor is charging or discharging, provided that positive current is taken as flowing in the direction indicated in Fig. 2. When the capacitor discharges, Kirchoff's loop rule for potentials gives

$$-IR - \frac{q}{C} = 0$$

which implies that

$$I = -\frac{q}{RC}. \tag{4}$$

Thus

$$\frac{dq}{dt} = -\frac{1}{RC}\,q. \tag{5}$$

Equation (5) is the fundamental equation that describes the behavior of this type of circuit; one of the aims of this experiment is to check whether this is correct.

## 1 Qualitative Investigations: Capacitor and Bulb

For typical small capacitances and resistances the charging process is very rapid and is not readily apparent to the eye. However, for the "supercapacitors" available in our lab, the process is briefly apparent. In this part of the experiment you will observe qualitatively a capacitor charging and discharging.

a) Connect the circuit of Fig. 3 using a "supercapacitor" and ensure that the capacitor is discharged by briefly connecting a wire across both terminals of the capacitor.

b) Close the switch and observe the brightness of the bulb as time passes. Based on your observations sketch a qualitative graph of the current through the bulb versus time.

c) Open the switch and discharge the capacitor. Close the switch and observe the ammeter reading versus time. Are your observations consistent with your graph from part 1.b)?

d) **Without discharging the capacitor,** remove the battery and reconnect the circuit as shown in Fig. 4. Close the switch and observe the ammeter reading and the bulb. Based on your observations sketch a qualitative graph of the current through the bulb versus time.

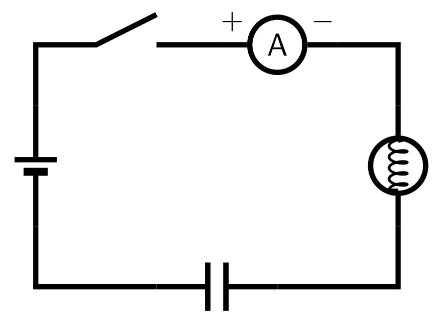


Figure 3: Capacitor and bulb in series: charging mode.

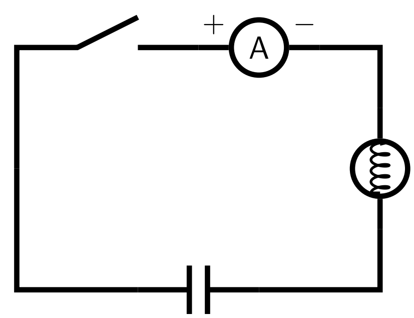


Figure 4: Capacitor and bulb in series: discharging mode.

## 2 Theory of a Discharging Capacitor

Although Eq. (5) describes of a discharging capacitor, it does not do so in terms of quantities which are easily measured. It can be manipulated to give predictions about measurable quantities by first solving it for the charge on the capacitor and then converting this to information about the potential difference across the capacitor.

a) The following is a candidate as a solution to Eq. (5):

$$q = q_0 \, e^{-t/\tau} \tag{6}$$

where $\tau$ is a constant, called the **capacitive time constant.** Substitute into Eq. (5) and determine an expression for $\tau$ in terms of $R$ and $C$.

b) For the capacitor $V_C = q/C$. Use this to show that the potential difference across the capacitor is

$$V_C = V_{C0}\, e^{-t/\tau} \tag{7}$$

where $V_{C0}$ is the potential difference across the capacitor at $t = 0$.

c) The previous part shows that the decay of the charge can be observed by measuring the potential across the capacitor. A plot of $V_C$ versus $t$ should yield a decaying exponential. The time constant $\tau$ could be extracted from this plot. However, a preferable alternative would be to *linearize the data* as follows. Starting with Eq. (7) determine an expression for $\ln(V_C)$ as a function of time. Suppose that a plot of $\ln(V_C)$ versus time was obtained. What should this graph look like? Describe *how* to obtain $\tau$ from the graph of $\ln(V_C)$ versus time.

## 3 Quantitative Investigations: Capacitor and Resistor

a) Measure and record resistance of the provided resistor. Record the capacitance of the provided capacitor. Connect the circuit shown in Fig. 2.

b) The potential difference across the capacitor, $V_C$, can be measured accurately as time passes using a voltage sensor connected to the PASCO interface. Open CapStone and set up the Voltage Sensor. Configure the display so that it will produce a graph of voltage versus time and a table of voltage versus time.

c) With the switch open, charge the capacitor by connecting the power supply directly across the capacitor's terminals. After a few seconds disconnect the power supply.

d) Close the switch, and run CapStone to record $V_C$ versus $t$. Print the graph of this. Does the graph appear to show exponential decay?

e) Copy the data from the table into Excel, plot the *linearized version* of the data (see Section 2). Print and save the plot. What does this plot ultimately say about Eq. (5)?

f) Use the plot to obtain the time constant. Compare this and your measured value for $R$ to obtain the capacitance of the capacitor. Compare this to the stated value of $C$.

## 4 Report

The lab exercise that you have done was broken down into many small steps, whose relationship to one another may not be obvious. In order to make sense of the entire exercise, compile a brief, informal report describing the aims, methods and results (for sections 2 and 3). This may be written in *bullet point form.* A *guideline* of what this might contain is:

- *Introduction*
  - Describe the aim of the experiment. What is the question that it addresses?
- *Set up and Theory*
  - Briefly describe/sketch the set up that can be used to meet the aim of the experiment.
  - Briefly describe what theory is useful for understanding the situation and what it eventually predicts (include equations that form predictions). Include derivations that were essential.
- *Experiment*
  - Provide details of the experimental set up.
  - Provide the experimental data and associated calculated quantities.
  - Provide the data analysis.
- *Conclusion and discussion*
  - Describe what the experiment showed. Did it verify something? If so, what? Did it answer a question that was posed earlier? If so, how?
  - Describe possible sources of error in this experiment. Be specific (stating that "human error" is an issue without describing what human error and how it entered is not acceptable). Describe, if possible, how such errors may be reduced.
  - Describe the main conclusion of this experiment. What answer does it give?