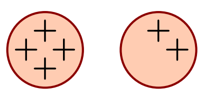
Phys 132: Supplementary Exercises

Electrostatics

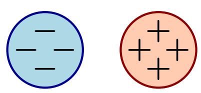
1 Charged spheres

Various identical metal spheres are separated and charged. The excess charges on each sphere, whose charges have the same *magnitude*, are illustrated. The spheres are supported by insulating stands and are brought into contact and then later are separated. Determine the charge on each sphere after they have been in contact. Briefly explain your answers.

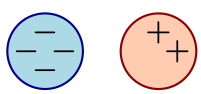
a) Before contact:



b) Before contact:

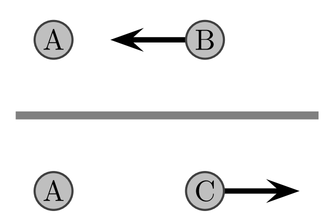


c) Before contact:



2 Pairs of objects

Pairs of various objects that may or may not be charged are placed near each other and the observed interactions are as illustrated. What interaction would occur if C were placed near to B? Explain your answer.



3 Interacting objects

Various objects, A, B, C and D, that may or may not be charged are placed near each other and the interactions are observed. It is observed that object A attracts object B, object A attracts object C and objects B and C do not interact. Object A repels object D.

a) Can one determine the types of charge of B and C with certainty? If so what are they?

b) Can one determine the types of charge of A and D with certainty? If so what are they?

c) Suppose that D is placed near to B. Explain how it will interact with B.

4 Sphere, rod and wire

A metal sphere is initially uncharged. A positively charged rod is held near to but not touching the sphere. At this time a wire is briefly connected from the ground to the side of the sphere opposite to the rod and is then removed, while the rod is in place. Subsequently the rod is then removed. After all of this a small negatively charged ball is held near to the sphere. Will the sphere exert a force on the ball? If so is it repulsive or attractive? Explain your answer. *Hint: ground can supply and absorb charged particles.*

5 Number of electrons

Copper consists of atoms that each have 29 electrons and mass 1.06×10^{-25} kg. Copper has a density of $8.69\,\mathrm{kg/m^3}$.

a) Determine the number of electrons in a cube of pure copper with sides of length 2.0 cm. Assume that the copper is electrically neutral.

b) Suppose that electrons are removed from the copper so that it has a net charge of 5.0×10^{-9} C (this is a typical number for capacitors in electronic circuits). Determine the number of electrons that have been removed.

c) What fraction of the total number of electrons in the copper were removed to give it charge 5.0×10^{-9} C?

6 Electric forces and charge magnitude

Two charged particles, A on the left and B on the right, are held at fixed locations. The charges on the particle can be adjusted while their locations stay fixed.

a) Suppose that the charge of B is tripled. By how many times does this increase the force that A exerts on B? Explain your answer.

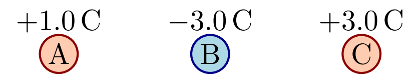
b) Suppose that the charge of B is tripled. By how many times does this increase the force that B exerts on A? Explain your answer.

c) Suppose that the charge of A is tripled and the charge of B is also tripled. By how many times does this increase the force that A exerts on B? Explain your answer.

d) Suppose that the charge of A is tripled and the charge of B is also tripled. By how many times does this increase the force that B exerts on A? Explain your answer.

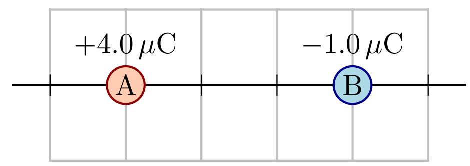
7 Electric forces and charges in a line

Three charged particles, A on the left, B in the center and C on the right, are held at fixed locations in a line. The distance between A and B is the same as between B and C. Rank the magnitudes of the net forces on each charge from smallest to largest. Explain your answer.



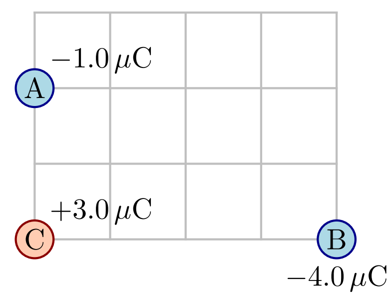
8 Linear charge arrangements and zero force

Two charged particles are held fixed as illustrated; the grid units are each 0.010 m. A third charge is placed along the axis that connects the two charges (either between the charges or beyond one of them). Determine the location at which the net force on this third charge are zero. *Hint: Set up the problem using axes with the origin at one of the charges and using algebraic coordinates for the location of the third charge.*



9 Two dimensional charge arrangements

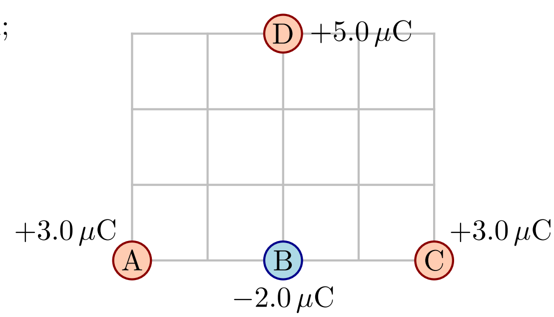
Three charged particles are held fixed as illustrated; the grid units are each 0.010 m. Determine the magnitude and direction of the net force on charge C.



10 Two dimensional charge arrangements

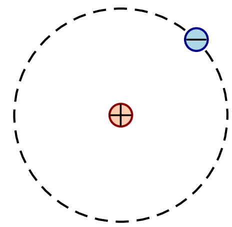
Four charged particles are held fixed as illustrated; the grid units are each 0.010 m.

a) Determine the net force on charge B.
b) Determine the net force on charge D.
c) Determine the net force on charge C.



11 Orbiting charge

Two particles have charges that are equal in magnitude but opposite in sign. The positive charge is held fixed and the negative charge is launched in such a way that it orbits the positive charge. Let Q be the magnitude of either charge, m the mass of the orbiting charge and r the radius of orbit.



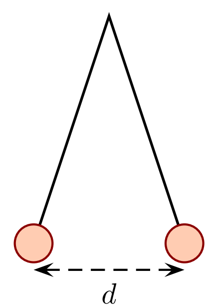
a) Starting with Newton's second law and using kinematics for uniform circular motion, determine an expression the speed of orbit v in terms of Q, m, r and constants.

b) Determine an expression for the time taken to complete one orbit, T in terms of Q, m, r and constants.

c) In a hydrogen atom the electron's orbital radius is approximately 10^{-10} m. Determine the time taken to complete one orbit.

12 Suspended charges

Two identical small metal spheres are each suspended from a string with length L. Each sphere is given the same charge q and they repel and reach an equilibrium state where they are a distance d apart. The mass of each sphere is m.



a) Determine an expression for q in terms of m, d, L and constants.

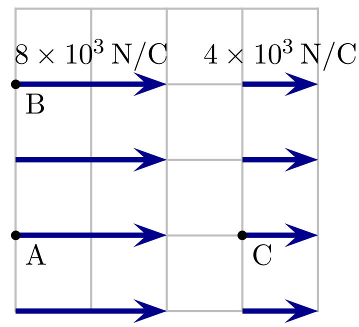
b) In a real case the mass of each ball is 8.0 g, each string is 25 cm long and the separation is 5.0 cm. Determine the charge on each ball.

13 Air breakdown field

Air breaks down (produces sparks and lightning) when the electric field reaches about 3.0×10^6 N/C. A metal sphere with radius 0.20 m carries a charge uniformly distributed on its surface and in this case, outside the sphere, the electric field is exactly the same as that produced if all the charge were concentrated at the center. Determine the charge that the sphere must carry so that the air at its surface breaks down.

14 Electric field at various locations

Hidden source charges produce the illustrated electric field.



a) Determine the force exerted on a proton that is placed at A. Determine the acceleration of a proton at this point.
b) Determine the force exerted on an electron that is placed at B. Determine the acceleration of an electron at this point.
c) Determine the force exerted on an electron that is placed at C. Determine the acceleration of an electron at this point.

15 Electric fields and forces

Two charged particles (sources) are held at fixed locations. A probe charge is placed at location A. Which of the following is true? Explain your answer.

i) The electric field at A is zero because the total charge is zero.

ii) The electric field at A points in one direction for a positive probe and the opposite for a negative probe; the force on the probe points in the same direction regardless of the probe.

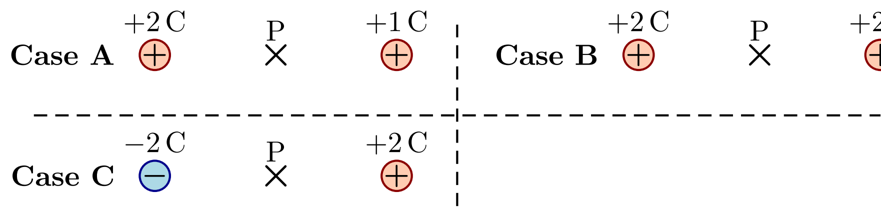
iii) The electric field at A points in one direction for a positive probe and the opposite for a negative probe; the force on the probe points one direction for a positive probe and the opposite for a negative probe.

iv) The electric field at A points in the same direction regardless of the probe charge; the force on the probe points in the same direction regardless of the probe charge.

v) The electric field at A points in the same direction regardless of the probe charge; the force on the probe points in different directions depending the probe charge.

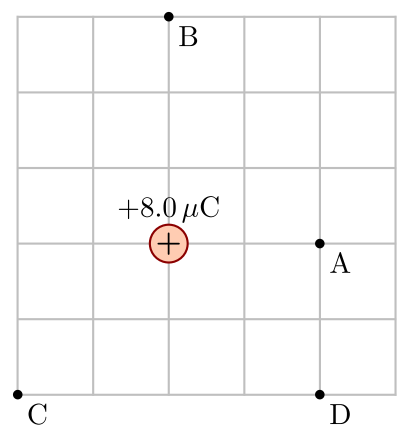
16 Electric field ranking

Three isolated pairs of charges are separated by the same distance in the three situations illustrated below. Rank the magnitudes of the electric fields at the midpoints (labeled P) from smallest to largest. Explain your answers.



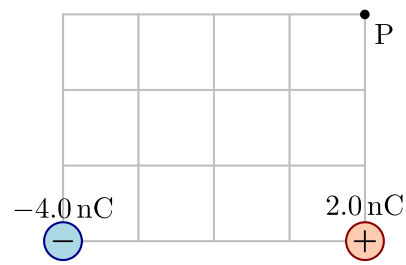
17 Electric field at various locations

A charged particle is held fixed as illustrated; the grid units are each 0.010 m. Determine the electric field vector at each point A,B,C and D. Draw the vectors on the diagram.



18 Electric field produced by two point charges

Two charged particles are held fixed as illustrated; the grid units are each 0.010 m. The aim of this exercise will be to determine the field at point P.



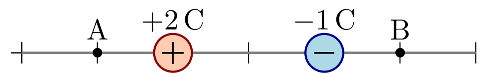
a) Indicate the directions of the electric fields produced by each source charge at point P.

b) Determine the magnitude of the electric field produced by each source charge at point P.

c) Using vector components add the two electric fields. Express the total electric field in terms of standard unit vectors.

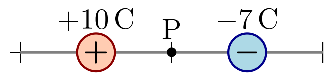
19 Electric fields at different locations

Two charged particles are held at fixed locations. Is the magnitude of the electric field produced by these charges at point A larger than, smaller than or the same as at point B? Explain your answer.



20 Electric fields produced by multiple charged particles and forces.

Two charged particles are held at fixed locations. Various probe charges are placed at the midpoint.



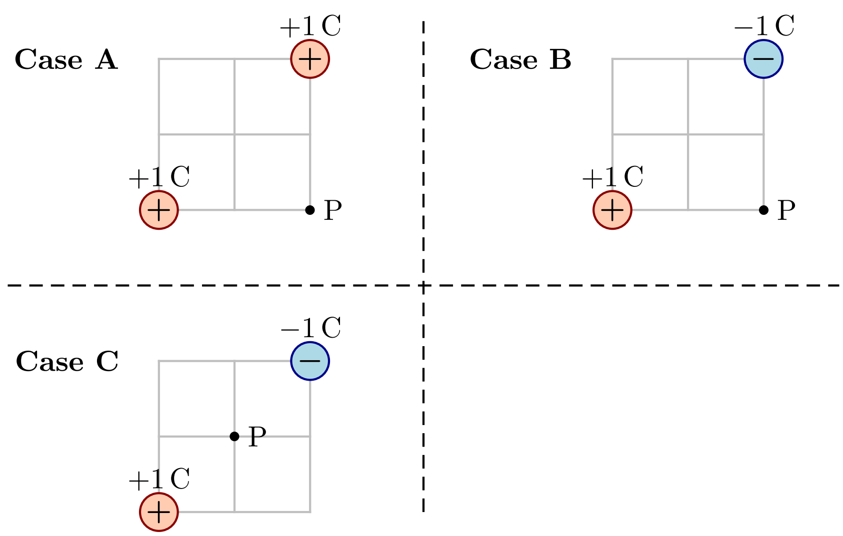
a) Does the direction and magnitude of the electric field produced by the two illustrated particles change when a positive probe at P is replaced by a negative probe at P?

b) Do the direction and magnitude of the force exerted on a probe placed at P change when a positive probe is replaced by a negative probe?

Explain your answers.

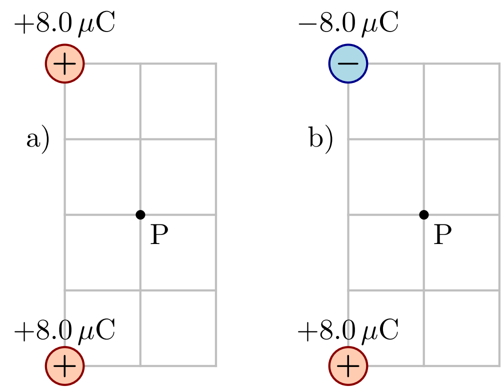
21 Electric fields in two dimensions

Three arrangements of two charged particles are as illustrated. Rank the electric fields in order of increasing magnitude, indicating equal cases where applicable. Explain your answer.



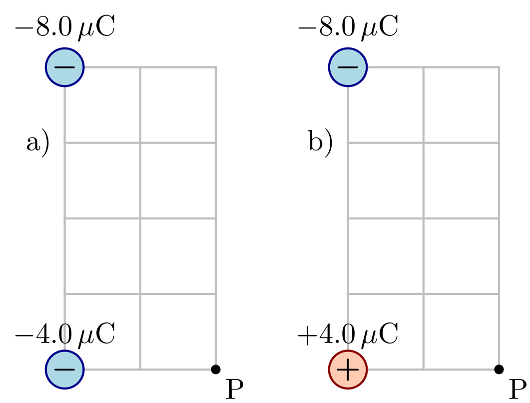
22 Electric field produced by two charges in two dimensions

In each of the following fixed charges are held fixed as illustrated. The grid units are each 0.020 m. Determine the electric field at point P in each case.



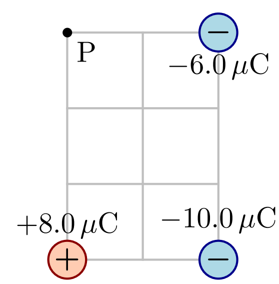
23 Electric field produced by two charges in two dimensions

In each of the following fixed charges are held fixed as illustrated. The grid units are each 0.020 m. Determine the electric field at point P in each case.



24 Electric field produced by multiple charges in two dimensions

Three charged particles are held fixed as illustrated; the grid units are each 0.020 m.



a) Determine the electric field at point P.

b) A particle with charge $+3.0\,\mu\text{C}$ is placed at P. Determine the force exerted on this particle by the other charged particles.

c) A particle with charge $-3.0\,\mu\text{C}$ is placed at P. Determine the force exerted on this particle by the other charged particles.

25 Carbon monoxide

Carbon monoxide is a molecule consisting of a single carbon and a single oxygen atom. The separation between these is roughly 1.0×10^{-10} m. The dipole moment of carbon monoxide is 4.07×10^{-31} C$\cdot$ m.

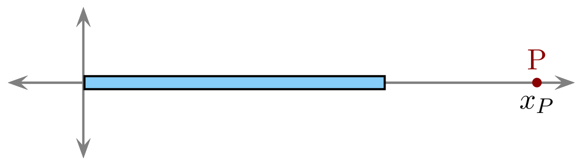
a) If the carbon monoxide molecule were to be modeled as a dipole consisting of a positive and a negative charge separated by 2.0×10^{-10} m, determine the magnitudes of these charges.

b) Determine the electric field along the dipole axis at a distance 20 nm from the center of the carbon monoxide molecule.

c) An electron is held at rest along the dipole axis at a distance 3.0×10^{-7} m from the center of the carbon monoxide molecule and is then released. Determine the acceleration of the electron at the moment it starts to move.

26 Field of a uniformly charged rod

A uniformly charged rod is oriented as illustrated. The length of the rod is L and the total charge (positive) is Q.



a) As an example, suppose that the length of the rod is $2.0\,\mathrm{m}$ and the total charge is $60\,\mathrm{C}$. Determine the charge density and use it to determine the charge in any segment of the rod with length $0.010\,\mathrm{m}$. Repeat this for a segment with length $0.0050\,\mathrm{m}$.

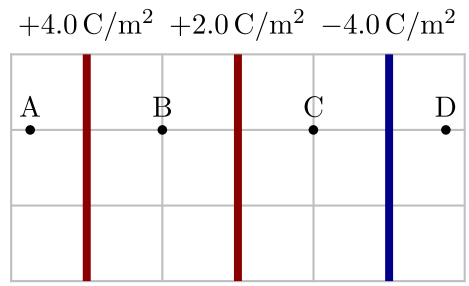
b) Consider a segment from $x \rightarrow x + \mathrm{d}x$ where x is some point in the rod. Write expressions for the length of the segment and the charge it contains (in terms of $\mathrm{d}x$ and λ).

c) Indicate the field produced by this segment and determine an expression for its components (in terms of $x_p, x, \mathrm{d}x$ and λ).

d) Write an expression (eventually in terms of calculus) for the components of the net electric field produced by all segments.

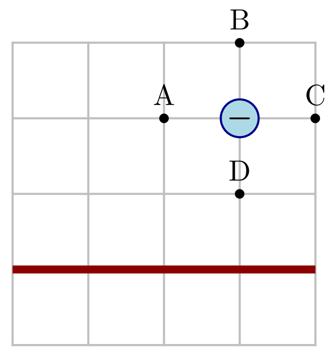
27 Electric fields produced by planes

Several parallel uniformly charged infinite planes, with indicated surface charge densities, are as illustrated. Rank the magnitudes of the electric fields at the indicated points, A, B, C and D. Explain your answer.



28 Electric field produced by point charge and plane

A point charge is held in the vicinity of a uniformly charged infinite plane as illustrated. The plane carries positive charge. Rank the magnitudes of the electric fields at the indicated points, A, B, C and D. Explain your answer.



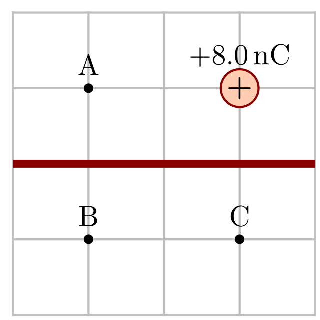
29 Electric field produced by point charge and plane

A point charge is held in the vicinity of a uniformly charged infinite plane as illustrated. The surface charge density on the sheet is $+1.5 \times 10^{-7}\,\mathrm{C/m^2}$. The grid units are each $0.040\,\mathrm{m}$.

a) Determine the net electric field at point A.

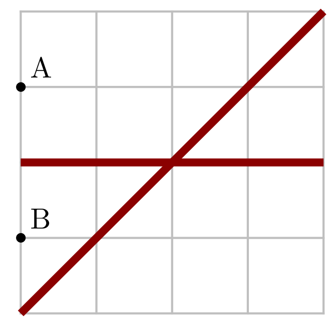
b) Determine the net electric field at point B.

c) Determine the net electric field at point C.



30 Electric fields produced by planes

Two uniformly charged infinite planes intersect at an angle of 45°, as illustrated. Each carries charge density $5.0 \times 10^{-6}\,\mathrm{C/m^2}$. The grid units are 0.010 m.



a) Determine the net electric field at point A.

b) Determine the net electric field at point B.

31 Particle above a uniformly charged infinite sheet

A horizontal infinite sheet is uniformly charged. A launcher fires an electron with speed $3.0 \times 10^{6}\,\mathrm{m/s}$ at an angle of 60° above the horizontal. The electron reaches a maximum height of 0.025 m above the plate.

a) Ignoring gravity, determine the electric field produced by the plate.

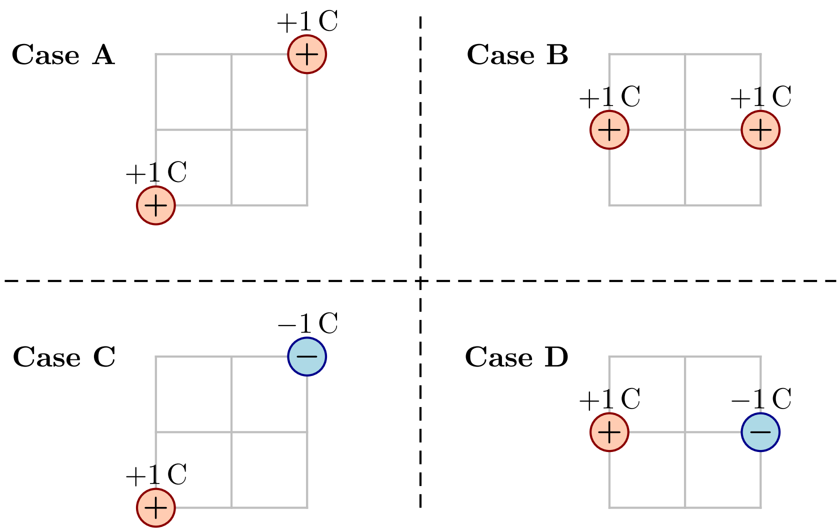
b) Determine the surface charge density of the plate.

c) Is it reasonable to ignore gravity? To answer this compare the magnitudes of the gravitational force on the electron to the electrostatic force on the electron.

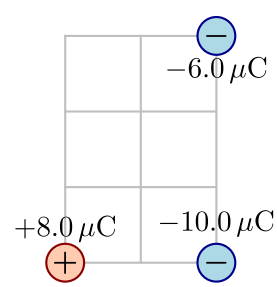
Electrostatic Potential

32 Electric potential energy for two charges

Four arrangements of two charged particles are as illustrated. Rank the these in terms of increasing electric potential energy, indicating equal cases where applicable. Explain your answer.



33 Electric potential energy for three charges



Three charged particles are held fixed as illustrated; the grid units are each 0.020 m.

a) Determine the electric potential energy of the arrangement.

b) Suppose that the $+8.0\,\mu\mathrm{C}$ charge were replaced by a $-8.0\,\mu\mathrm{C}$. Would the electric potential energy increase, decrease or stay constant? Explain your answer.

34 Escape speed of a charge

A small 0.040 m diameter sphere carries charge +20.0 nC. A proton is released from rest at the surface of the sphere.

a) Determine the speed with which the proton travels when it is infinitely far from the sphere.

b) If the radius of the sphere were decreased while the charge stays constant would the ultimate speed of the proton be larger, smaller or the same as before? Explain your answer.

35 Alpha particle and a gold nucleus

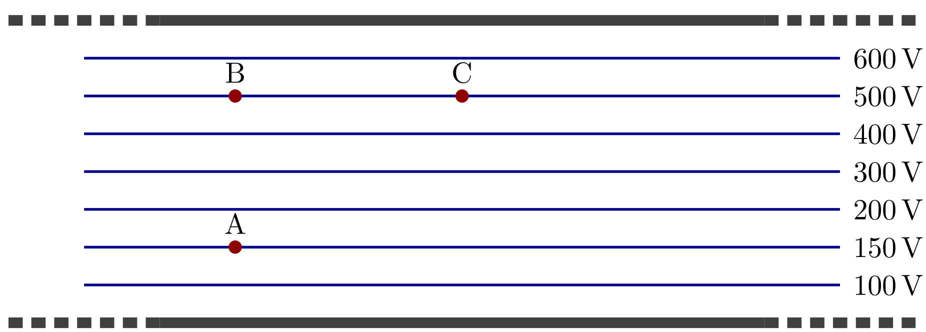
An alpha particle consists of two protons and two neutrons. Suppose that an alpha particle is directly toward a nucleus of a gold atom (the nucleus contains neutrons and 79 protons). Assume that the alpha particle is fired with speed 3.0×10^6 m/s. This speed is sufficient that the force exerted by the electrons can be ignored to same extent; assume that it is negligible.

a) If the alpha particle is fired from infinitely far away, determine the closest that it approaches the center of the nucleus.

b) Clearly the alpha particle cannot be fired from infinitely far away. Assume that it is fired from 2.0 m with the same speed toward the nucleus. Determine the closest that it approaches the center of the nucleus. Does this differ much from the result if it were fired from infinitely far away?

36 Electrostatic potential between plates

Two parallel plates are charged. The electrostatic potential is such that it is constant along lines that are parallel to the plates. Several of these are as illustrated. Positively charged particles are moved between the two points.



a) A charge is released at A with an initial velocity in the vertical direction and moves to B in a straight line. In a separate experiment the same charge is released at A with velocity of the same magnitude, but angled toward C. It moves in a curved path arriving at C. How does the change in electrostatic potential energy in the A $\rightarrow$ B case compare to that of the A $\rightarrow$ C case? How does the final speed in the A $\rightarrow$ B case compare to that of the A $\rightarrow$ C case?

b) The particle is made to move in two paths from B to C at a constant speed (an external force will be required to do this). One path is a straight line from B to C and the other a semicircular path within the plates from B to C. Compare the changes in total energy and the net work done by the external force in the two cases. Do your answers depend on the path taken? Would they be valid for any path taken?

37 Accelerating electrons

Electrons can be made to accelerate by creating an electrostatic potential difference across a region of space.

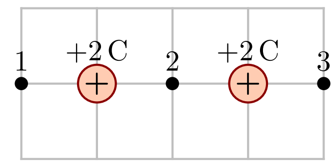
a) Suppose that an electron is initially at rest and moves through a region where the potential difference is 100 V. Determine the speed of the electron at the end of this region.

b) Through what potential difference must an electron be accelerated so that it reaches a speed of 1.5×10^7 m/s if it was initially at rest?

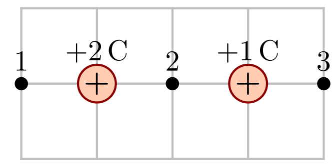
38 Electric potential for two charges

Various arrangements of two charged particles are as illustrated. In each case rank the potentials at the indicated points in order from smallest to largest. Explain your answer.

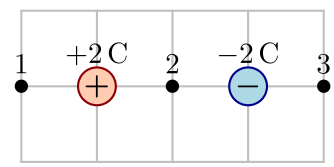
Case a)



Case b)



Case c)



39 Electric potential energy and motion

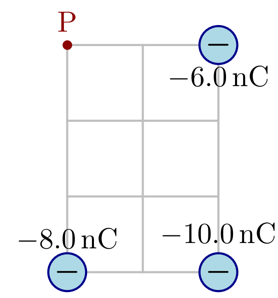
The electric field between the plates of a parallel plate capacitor is 4.0×10^4 N/C. A proton is released from rest at a point between the plates.

a) Determine the change in electric potential energy of the proton after it has moved 0.0025 m.

b) Determine the speed of the proton after it has moved 0.0025 m for the point where it was released.

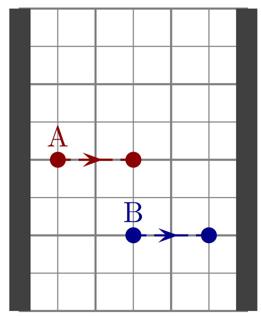
40 Escape speed of a particle

A 0.0050 kg sphere with charge −0.0080 C is held at rest at the point labeled P. The grid units are 0.010 m. The sphere is release. Determine the speed of the sphere when it is infinitely far from the illustrated charges.



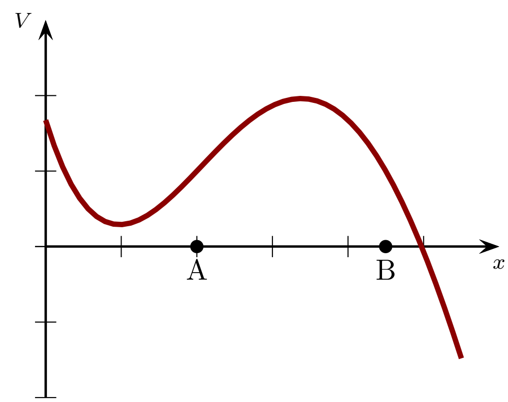
41 Protons in a capacitor

Two protons are released from rest at different points inside a parallel plate capacitor. Assume that the plates are infinite. The right plate of the capacitor is negatively charged, the left plate positively. They travel as illustrated. How does the speed of A compare (smaller, larger, or same as) to that of B after each has traveled the same distance? Explain your answer.



42 Particles in a potential

Hidden source charges produce the illustrated potential. Various probe charges are, in separate experiments, released from rest at the indicated locations.



a) A positive charge is released from A. In which direction and how far will it move?

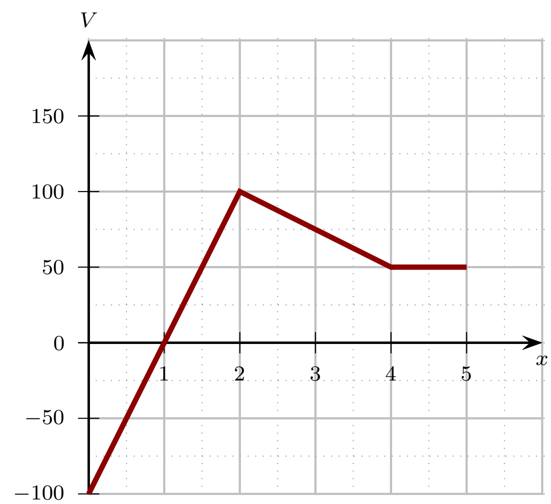
b) A positive charge is released from B. In which direction and how far will it move?

c) A negative charge is released from A. In which direction and how far will it move?

d) A negative charge is released from B. In which direction and how far will it move?

43 Particles in a potential

Various metal plates that are perpendicular to the x axis produce the illustrated electric potential. The horizontal axis units are meters and the vertical axis units are Volts.



a) A sphere with mass 0.020 kg and charge -8.0×10^{-2} C is released from rest at $x = 0$ m. Determine its speed when it is at $x = 2.0$ m and again at $x = 5.0$ m.

b) A sphere with mass 0.40 kg and charge $+7.5 \times 10^{-2}$ C, moves left at $x = 5$ m with speed 5.0 m/s. Determine its subsequent minimum speed.

44 Parallel plate capacitor and charge

A parallel plate capacitor has two square plates with sides 0.04 m. These are placed a distance 2.5×10^{-3} m apart. These carry equal but opposite charges. Assume that the charges are uniformly distributed (on each plate) and that the plates can be regarded as infinite. Suppose that the electric potential difference from one plate to the other is 12 V.

a) Determine the charge per surface area on each plate.

b) Determine the charge on each plate.

45 Parallel plate capacitor and electron acceleration

A parallel plate capacitor has two circular plates with radius 0.125 m. These are placed a distance 5.0×10^{-3} m apart. These carry equal but opposite charges with magnitudes 45 nC. Assume that the charges are uniformly distributed (on each plate) and that the plates can be regarded as infinite.

a) Determine the charge density (charge per area) on each plate.

b) Determine the electric potential difference from one plate to the other.

c) An electron is held at rest on the negative plate. Determine its speed just before it hits the positively charge plate.

46 Electric potential produced by a line of charge

An infinite line of charge lies along the z axis. The linear charge density (charge per meter) is λ. The electric field produced by the charge points radially outwards and along the x-axis it is

$$\vec{\mathbf{E}} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x}.$$

Suppose that $\lambda = +7.5 \times 10^{-9}\,\mathrm{C/m}$.

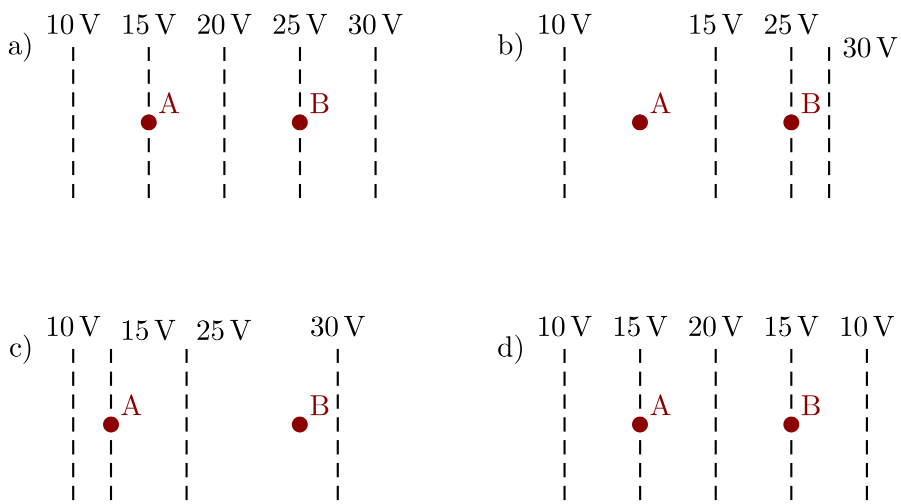
a) Determine the electric potential difference from $x = 0.5\,\mathrm{m}$ to $x = 1.0\,\mathrm{m}$.

b) If a proton were released at $x = 0.5\,\mathrm{m}$, determine its speed at $x = 1.0\,\mathrm{m}$.

c) Determine the electric potential difference from $x = 1.0\,\mathrm{m}$ to $x = 1.5\,\mathrm{m}$.

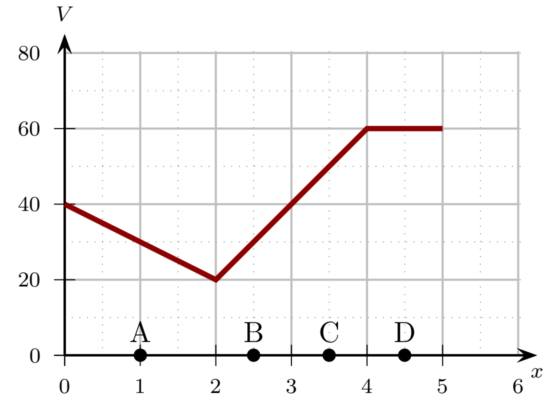
47 Equipotentials and fields

Each of the illustrations shows a set of equipotentials produced by hidden charges. Describe whether the magnitude of the electric field at A is large than, smaller than or the same size as that at B. Explain your answers.



48 Potentials and fields

Various metal plates that are perpendicular to the x axis produce the illustrated electric potential. The horizontal axis units are meters and the vertical axis units are Volts. Determine the electric field at points A,B,C and D.



49 Potential within a charged sphere

A solid sphere with radius R is uniformly charged with total charge Q. With the origin at the center of the sphere, electric potential inside the sphere in the xy plane is

$$V = -\frac{Q}{8\pi\epsilon_0}\frac{x^2+y^2}{R^3}.$$

a) Describe the shape of any equipotential inside the sphere. Sketch several of these as accurately as possible.

b) Use the equipotentials to describe the direction of the electric field inside the sphere.

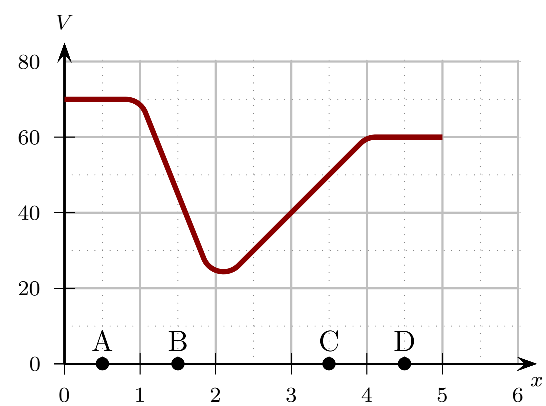
c) Use the potential to determine an expression for the electric field anywhere along the x axis.

50 Potentials and motion

Various metal plates that are perpendicular to the x axis produce the illustrated electric potential. The horizontal axis units are meters and the vertical axis units are Volts. In separate experiments, electrons are released from rest at the indicated locations, A, B, C, and D.

a) At each location, describe whether the electron moves and, if so, in what direction.

b) Rank the magnitudes of the accelerations of the electrons that are placed at the four locations.

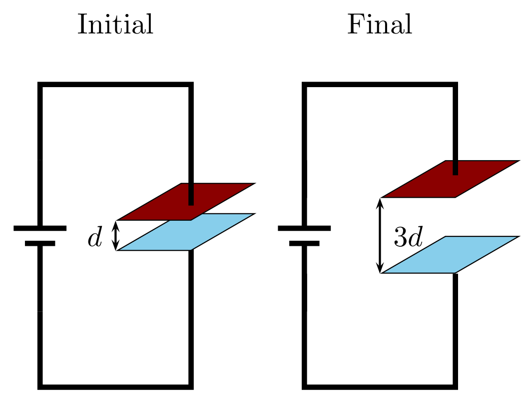


Capacitors

51 Parallel plate capacitors

Suppose that you aim to construct a 20 nF capacitor from two circular disks that are spaced 0.50 mm apart. Determine the radius of the plates that will accomplish this.

52 Capacitors and energy



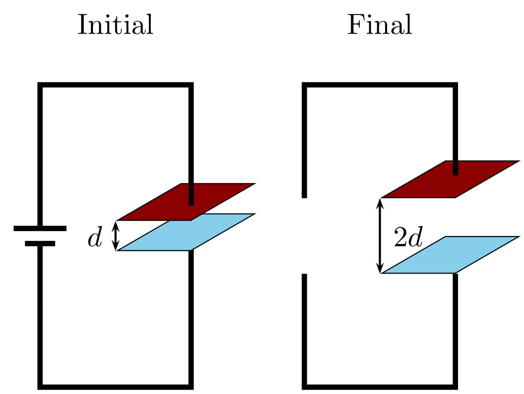
A parallel plate capacitor is connected to a battery that provides electric potential difference ΔV. The capacitor is allowed to charge completely. While the capacitor is connected to the battery, the plates are separated so that the distance between them is three times what it had been.

a) Does the electric potential difference across the capacitor plates change from the beginning to the end of this process? If so by what factor? Explain your answer.

b) Does the charge stored on the capacitor plates change from the beginning to the end of this process? If so by what factor? Explain your answer.

c) Does the energy stored in the capacitor change from the beginning to the end of this process? If so by what factor? Explain your answer.

53 Capacitors and energy



A parallel plate capacitor is connected to a battery that provides electric potential difference ΔV. The capacitor is allowed to charge completely. The battery is disconnnected. After this has occured, the plates are separated so that the distance between them is twice times what it had been. Does the energy stored in the capacitor change from the beginning to the end of this process? If so by what factor? Explain your answer.

54 Capacitors in parallel

Two capacitors, one with capacitance 30 nF and the other 50 nF are connected in parallel to a 15 V battery.

a) Determine the charge stored in each capacitor.

b) Determine the energy stored in each capacitor.

c) Determine the total charge stored on both capacitors and the total energy stored in both.

55 Capacitor and dielectric

A parallel plate capacitor is attached to a battery and then disconnected.

a) After the battery is disconnected, a dielectric is inserted between the plates. Does the charge on the plates remain constant, increase or decrease while the dielectric is being inserted? Does the potential difference across the plates remain constant, increase or decrease while the dielectric is being inserted? Explain your answer.

b) Does the energy stored in the capacitor remain constant, increase or decrease while the dielectric is being inserted? Explain your answer.

c) Does the capacitor exert a force on the dielectric while it is being inserted? If so, in which direction? Explain your answer.

A parallel plate capacitor is attached to a battery and is left connected.

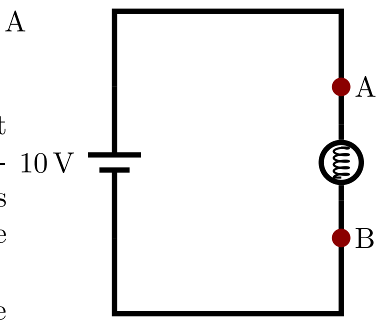
d) While the battery is connected, a dielectric is inserted between the plates. Does the charge on the plates remain constant, increase or decrease while the dielectric is being inserted? Does the potential difference across the plates remain constant, increase or decrease while the dielectric is being inserted? Explain your answer.

e) Does the energy stored in the capacitor remain constant, increase or decrease while the dielectric is being inserted and the battery is connected? Explain your answer.

f) Does the capacitor exert a force on the dielectric while it is being inserted and the battery is connected? If so, in which direction? Explain your answer.

Currents and Circuits

56 Energy and currents



A battery and bulb are connected in the illustrated circuit. A steady current of 2.0 A flows through the bulb.

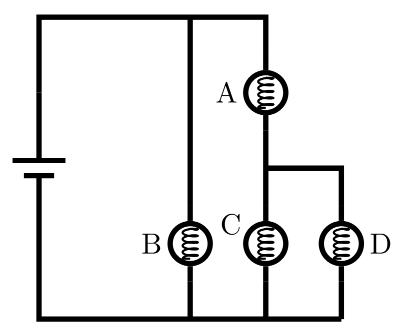
a) Assume that positive charge moves from point A to point B. Determine the charge that passes point B in 5.0 s. Determine the change in electrostatic potential energy of this charge as it moves from point A to point B. Does the charge gain or lose electrostatic energy?

b) Using the same data as above, assume that negative charge moves from point B to point A. Determine the change in electrostatic potential energy of this charge as it moves from point B to point A. Does the charge gain or lose electrostatic energy?

c) Suppose that the battery can provide 9.6 kJ of energy. Determine the amount of time for which the battery could operate in this circuit.

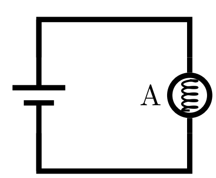
57 Currents at junctions

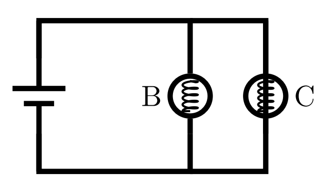
A battery and bulbs are connected in the illustrated circuit. The current in C is the same as in D. Suppose that the current in A is 6 A and the current though the battery is 10 A. Determine the currents through the other bulbs.



58 Currents in bulbs

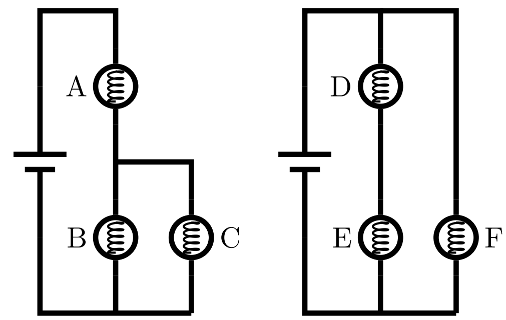
Various identical bulbs are connected to identical batteries in the illustrated circuits. Rank bulbs A, B and C in order of increasing brightness. Explain your answer.





59 Currents in bulbs

Various identical bulbs are connected to identical batteries in the illustrated circuits.



a) Rank the bulbs A, B, and C in order of increasing brightness. Explain your answer.
b) Rank the bulbs D, E, and F in order of increasing brightness. Explain your answer.

60 Air conditioner power

A window unit air conditioner uses power 1400 W and has a thermostat that turns it on and off. During the summer it runs for 9.0 hr every day. Th air conditioner is connected to the 120 V mains outlet.

a) Determine the total energy consumed by the air conditioner in a day.
b) Determine the electrical current that flows into the air conditioner as it runs.

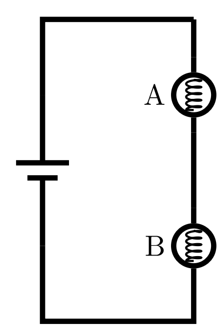
61 Toaster resistance

A toaster provides power 1200 W when it is connected to a 120 V mains outlet.

a) Determine the resistance of the toaster.
b) Suppose that the toaster is connected to a 240 V outlet. Determine the power that it provides, assuming that its resistance stays constant.

62 Bulbs in series

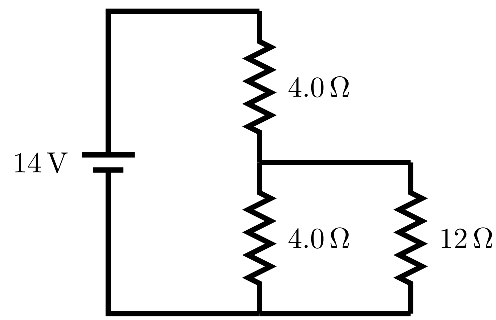
A battery and bulbs are connected in the illustrated circuit. The resistance of A is three times the resistance of B.



a) Determine the ratio of the currents through the bulbs, I_B/I_A.
b) Determine the ratio of the potential differences across the bulbs, $\Delta V_B/\Delta V_A$.
c) Determine the ratio of the powers dissipated by the bulbs, P_B/P_A.

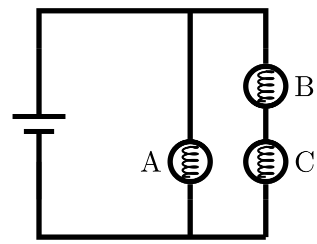
63 Resistors in series and parallel

Determine the currents through, voltages across and the power dissipated by each resistor in the illustrated circuit.



64 Currents in bulbs

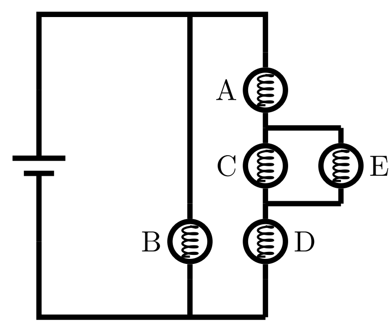
A battery and identical bulbs are connected in the illustrated circuit. Rank the bulbs in order of increasing brightness.



a) Rank the bulbs in order of increasing brightness. Explain your answer.
b) Suppose that bulb C is replaced by a wire. Describe the effect of this change on the brightness of bulb A and B. Explain your answer.

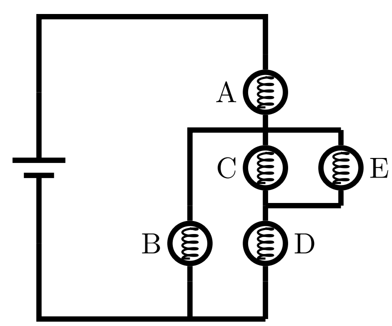
65 Currents in bulbs

A battery and identical bulbs are connected in the illustrated circuit. Rank the bulbs in order of increasing brightness. Explain your answer.



66 Currents in bulbs

A battery and identical bulbs are connected in the illustrated circuit. Rank the bulbs in order of increasing brightness. Explain your answer.

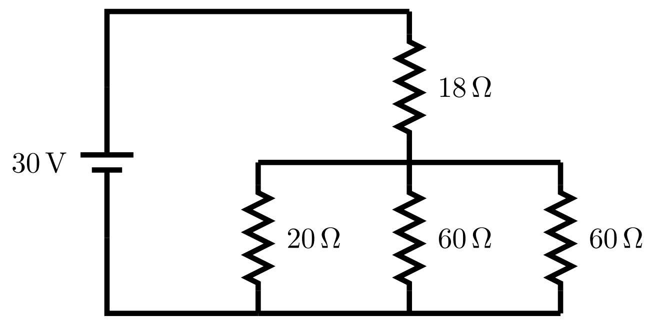


67 Resistor combinations

You are given three $4\,\Omega$ resistors and can connect them in various ways. List all possible effective resistances that you can attain by combining all three resistors.

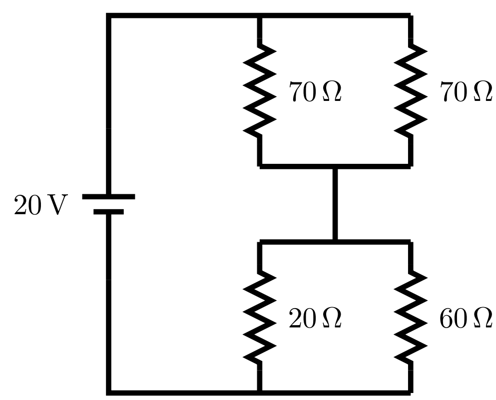
68 Resistors in series and parallel

Determine the currents through, voltages across and the power dissipated by each resistor in the illustrated circuit.



69 Resistors in series and parallel

Determine the currents through, voltages across and the power dissipated by each resistor in the illustrated circuit.



70 Real batteries

A real battery with internal resistance r and EMF $\mathcal{E}$ is connected to a resistor with resistance R.

a) Explain whether the voltage across the (external) resistor increases, decreases or stays constant as the external resistance increases.

b) Show that the power provided to the external resistor is

$$P = \frac{R}{(r+R)^2}\,\mathcal{E}^2.$$

c) How much power is delivered to the external resistor as $R \to 0$?

d) How much power is delivered to the external resistor as $R \to \infty$?

e) Determine the value of the external resistance so that the power delivered to this is a maximum.

71 Discharging capacitor

A capacitor with capacitance, C is connected to a resistor with resistance R. At time $t = 0$ the charge on the capacitor is Q_0. Subsequently the charge satisfies

$$\frac{dQ}{dt} = -\frac{1}{RC}Q. \tag{1}$$

The aim of this exercise is to solve this for Q at all later times.

a) A solution to Eq. (1) is a function $Q(t)$ such that when it is substituted into both sides of equation Eq. (1) the resulting statement is *true for all times t.* To test this consider the possibility $Q(t) = t^3$. Substitute into both sides of Eq. (1) and carry out the relevant mathematics. Is the resulting equality *true for all times t?*

b) Consider the possible solution $Q = Ae^{\alpha t}$ where A and α are constants. Substitute into both sides of Eq. (1) and carry out the relevant mathematics. Is the resulting equality *true for all times t?* What condition on α would guarantee this?

c) Determine a value for A such that the charge at $t = 0$ is Q_0.

d) The resulting expression is written as

$$Q = Q_0 e^{-t/\tau} \tag{2}$$

where τ is a constant called the capacitive time constant. Determine an expression for τ in terms of R and C.

The capacitive time constant quantifies the rate at which the capacitor charge decays. In the following, suppose that $R = 2.0 \times 10^5\,\Omega$ and $C = 5.0 \times 10^{-7}\,\text{F}$.

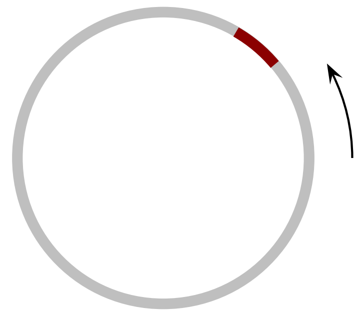
e) Determine the capacitive time constant. What fraction of the initial charge remains at $t = 0.1\,\text{s}$? What fraction of the initial charge remains at $t = 0.2\,\text{s}$?

f) In experiments data for Q versus t is conveniently represented by plotting $\ln(Q)$ versus t. Determine an expression for $\ln(Q)$ in terms of t and constants. What sort of plot would this yield? What is the slope of this plot?

Currents and Magnetic Fields

72 Magnetic field at the center of a circular current loop

A circular loop with radius R carries current I as illustrated. The aim of this exercise is to use the Biot-Savart law to determine the magnetic field at the center of the loop. First consider the contribution to the field from the shaded section. This is



$$\vec{\mathbf{B}}_{\text{segment}} = \frac{\mu_0}{4\pi} I \frac{\Delta\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}. \tag{3}$$

a) Use the diagram to indicate the vectors $\vec{\mathbf{r}}, \hat{\mathbf{r}}$ and $\Delta\vec{\mathbf{s}}$.

b) Determine the direction of $\Delta\vec{\mathbf{s}} \times \hat{\mathbf{r}}$. Would the direction be different for a different segment of the current in the loop?

c) Let the magnitude of $\Delta\vec{\mathbf{s}}$ be Δs. Determine an expression for the magnitude of $\Delta\vec{\mathbf{s}} \times \hat{\mathbf{r}}$ and use this and Eq. (3) to determine an expression for the magnitude of $\vec{\mathbf{B}}_{\text{segment}}$.

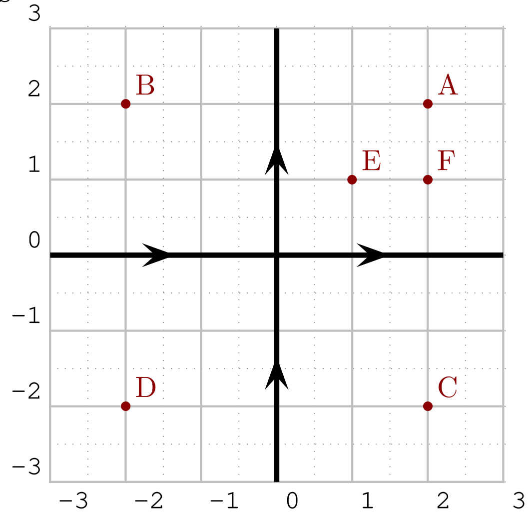
d) What is the value of r in this situation? Use your answer to determine an expression for the magnitude of $\vec{\mathbf{B}}_{\text{segment}}$. .

e) Add the contributions from all segments to obtain an expression for the field at the center of the loop.

f) Suppose that you aimed to use this to determine the magnetic field at a point that is not at the center of the loop. What mathematical difficulties would complicate the calculation compared to that for the center of the loop?

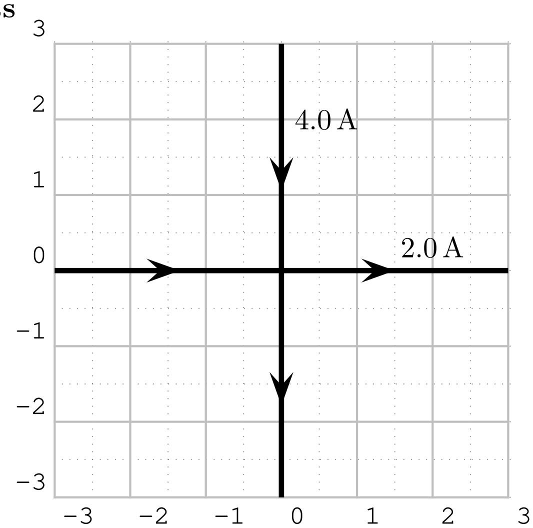
73 Field produced by pairs of straight currents

Two infinitely long wires are oriented as illustrated and each carry the same current. Rank the magnitudes of the net magnetic fields produced at the indicated points. Explain your answer.



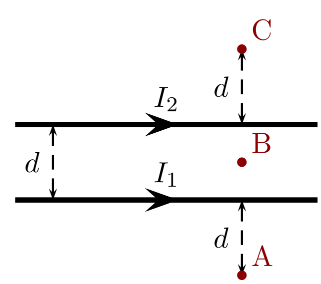
74 Field produced by pairs of straight currents

Two infinitely long wires are oriented as illustrated and carry currents as illustrated. Indicate the locations at which the net magnetic field is zero. Explain your answer.



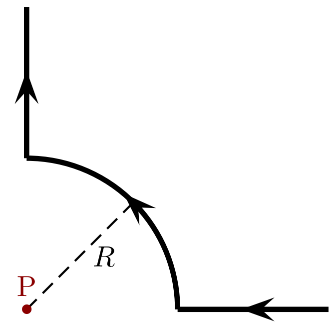
75 Magnetic fields produced by parallel currents

Two infinitely long straight parallel wires carry the illustrated currents with $I_1 > I_2$. Determine an expression for the magnetic field at the points A, B (midway between the wires) and C.



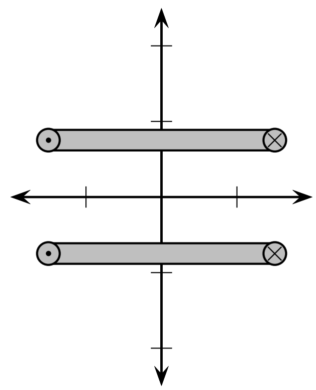
76 Magnetic fields produced by a current

Two infinitely long straight wires make an arc of radius R. Determine an expression for the magnetic field at point P.



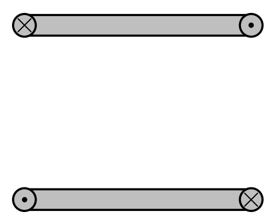
77 Fields produced by a Helmholtz coil

Two current loops are situated as illustrated; both are perpendicular to the plane of the page. The distance between the loops is exactly equal to the radius of each loop and the loops carry the same current. This combination is called a Helmholtz coil. Determine an expression for the field produced by the two coils at the point midway between them and along the axis that connects them.



78 Forces exerted by one current loop on another.

Two current loops are situated as illustrated; both are perpendicular to the plane of the page.



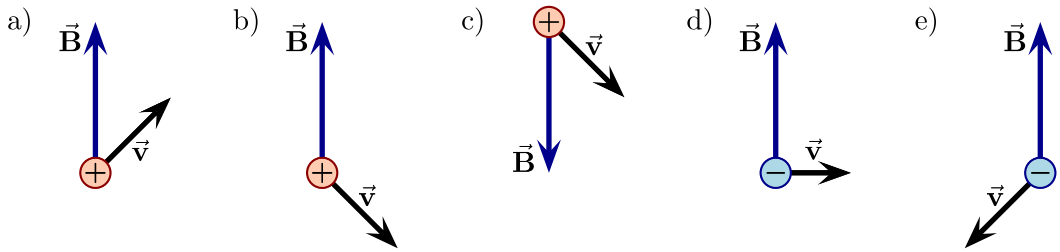
a) Indicate the direction of the magnetic dipole moment produced by each loop.

b) Use the magnetic dipole moments to describe the direction of the force exerted by one loop on the other.

Magnetic Forces

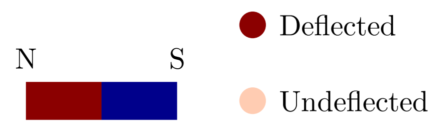
79 Force on a charged particle

Each of the following illustrates a charged particle moving in an external magnetic field. Indicate the direction of the force exerted by the magnetic field on the charged particle for each case. Explain your answer.



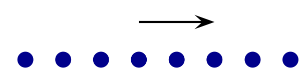
80 Force on a beam of particles

A beam of particles is fired at high speed and travels in a straight line. You view the beam traveling toward you. A magnet is held as illustrated and the beam is deflected as illustrated. What charge do the particles have? Explain your answer.



81 Force on a beam of electrons

A beam of electrons moves from left to right as illustrated. The electrons are placed in a uniform magnetic field.



a) If initially the electrons are to be deflected downwards, what is the direction of the magnetic field? Explain your answer.

b) As the electrons continue to move in the uniform field, does the magnitude of the force on the electrons increase, decrease or stay the same? Explain your answer.

c) As the electrons continue to move in the uniform field, does the speed of the electrons increase, decrease or stay the same? Explain your answer.

82 Mass spectrometer

The set up of a mass spectrometer is illustrated in Fig P29.64 (page 834). Suppose that the particles are accelerated from rest through a potential difference of ΔV. Show that the radius of orbit is of a particle with charge q is

$$r = \frac{1}{B}\sqrt{\frac{2m|\Delta V|}{|q|}}$$

where B is the magnitude of the field and m is the mass of the particle.

83 Cyclotron

In a cyclotron a charged particle orbits in a circle perpendicular to a known uniform magnetic field. Suppose that the magnetic field strength is B, the charge of the particle is q, its mass is m and the radius of orbit is R.

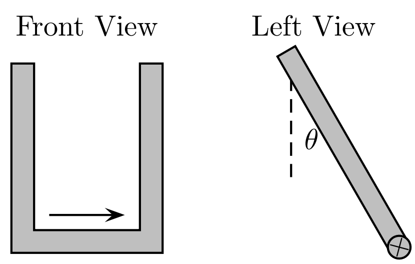
a) Starting with and using Newton's Second Law determine an expression for the time taken to complete on orbit, T, in terms of B, q and m. This is called the period of orbit.

b) The frequency of orbit is $f := 1/T$. Determine an expression for f in terms of B, q and m.

c) The charged particle will radiate an electromagnetic signal at frequency f which is easily measured. Suppose that the magnetic field strength is known. Explain how a cyclotron could be used to determine the charge to mass ratio q/m for the particle.

84 Force on a current beam

A current beam is shaped as shown and carries the illustrated current. The beam can pivot about its two upper points. The beam is placed in a uniform vertical magnetic field which causes it to deflect and remain stationary at an angle of θ with respect to the vertical.

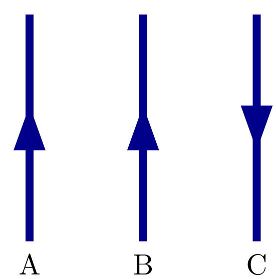


a) Suppose that the length of the horizontal section is l, the current is I, the mass of the entire beam is m and the magnetic field has magnitude B. Determine an expression for θ in terms of these quantities.

b) In a lab the beam has mass 10 g and length 15 cm and the power supply provides a maximum current of 3.0 A. Determine the magnetic field strength needed to deflect the beam at an angle of 5.0° from the vertical.

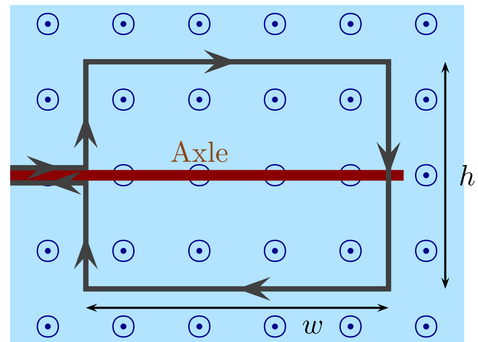
85 Forces on wires

Three wires each carry currents of identical magnitudes, I in the illustrated directions. The distance between adjacent wires, d are equal. Determine an expression for the net force on each wire and rank the wires in order of the magnitudes of the net magnetic force on each.



86 Current loop in a uniform magnetic field

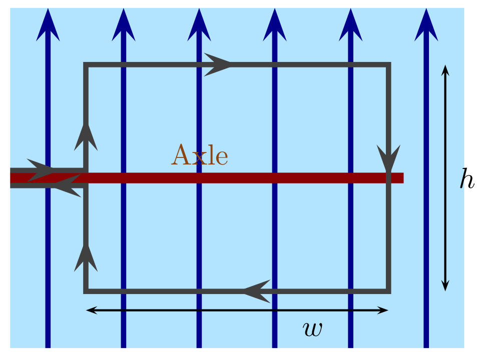
A loop is placed in a uniform magnetic field and a current flows as illustrated. The magnetic field strength is B and the magnitude of the current is I. Initially the loop lies perpendicular to the magnetic field as illustrated.



a) For the initial configuration, determine an expression for the force on each side of the loop. Determine an expression for the net force on the loop.

b) For the initial configuration, determine an expression for the torque (about the axle) on each side of the loop and the net torque on the loop.

Suppose that the loop lies in the plane of the magnetic field as illustrated.

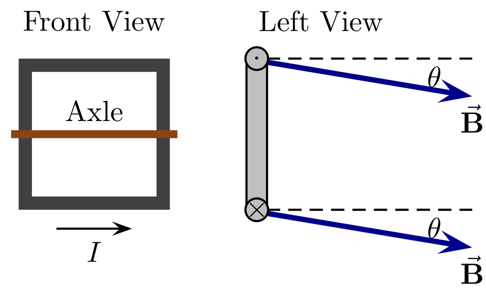


c) In this configuration, determine an expression for the force on each side of the loop. Determine an expression for the net force on the loop.

d) In this configuration, determine an expression for the torque (about the axle) on each side of the loop and the net torque on the loop. Rewrite the expression in terms of the area of the loop.

e) Describe how the loop would begin to move if it were released from this position.

87 Current loop in a uniform magnetic field

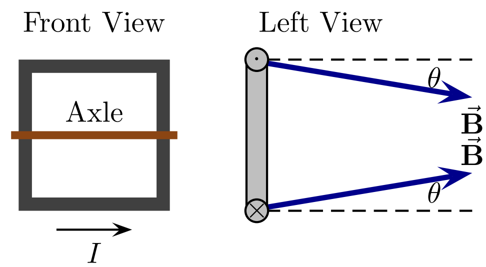


A square loop with sides of length L carries current I is placed in a uniform magnetic field with magnitude B. Viewed from the left side the field lies in the plane of the page and is angled as illustrated. Viewed from this side the current in the upper end of the loop is out of the page and in the lower end of the loop it is into the page.

a) Determine an expression for the net force that acts on the loop in terms of L, I, B and θ.

b) Determine an expression for the net torque on the loop (about an axle that is out of the page in the left view).

88 Current loop in a non-uniform magnetic field



A square loop with sides of length L carries current I is placed in a non-uniform magnetic field with magnitude B. Viewed from the left side the field lies in the plane of the page and is angled as illustrated. Viewed from this side the current in the upper end of the loop is out of the page and in the lower end of the loop it is into the page.

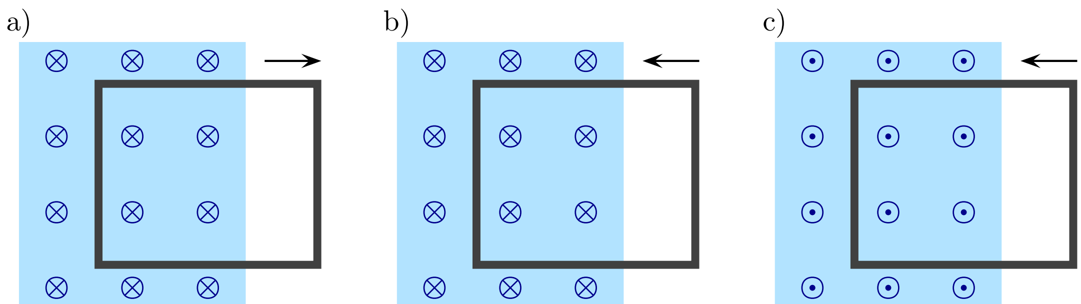
a) Determine an expression for the net force that acts on the loop in terms of L, I, B and θ.

b) Is the net torque on the loop (about an axle that is out of the page in the left view) zero or non-zero? Explain your answer.

Magnetic Induction

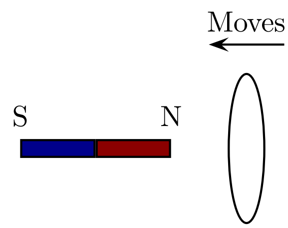
89 Motional EMF and current

In each of the following a loop lies partly in a uniform magnetic field and is dragged as illustrated. In each case, determine if the dragging produces any current and, if so, its direction. Explain your answers.



90 Loop moved near a permanent magnet

A circular loop is initially held at rest near to a bar magnet. The bar magnet lies along the axis of the loop. The loop is then pushed toward the magnet. In each of the following explain your answers.



a) While the loop is initially at rest, is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

b) While the loop is initially at rest, does the magnet exert a force on the loop? If so, what is its direction?

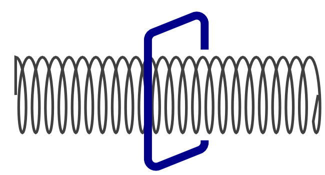
c) While the loop approaches the magnet, is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

d) While the loop approaches the magnet, does the magnet exert a force on the loop? If so, what is its direction?

e) The loop eventually passes "across" the magnet and moves left away from the magnet, During this period is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)? Does the magnet exert a force on the loop? If so, what is its direction?

91 Square loop around a solenoid

A square loop is placed around a solenoid, which can be approximated as infinitely long. The square loop and the solenoid are arranged so that their axes are along the same line. When viewed from the right, the current in the solenoid flows clockwise. In each of the following explain your answers.

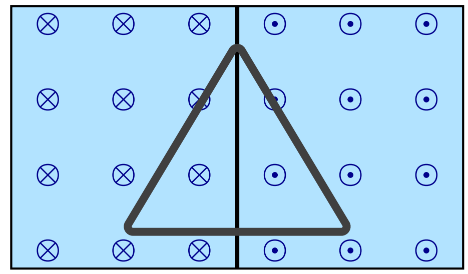


a) Suppose that the current in the solenoid is constant. Is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

b) Suppose that the current in the solenoid is increasing. Is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

c) Suppose that the current in the solenoid is decreasing. Is there a current in the loop? If so, what is its direction (as viewed from the right edge of the diagram)?

92 Triangular loop in a magnetic field



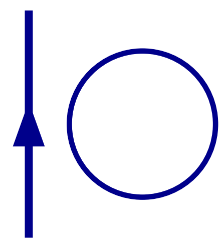
The illustrated regions each have a uniform magnetic field with the same magnitude but opposite directions. A triangular loop is initially at the illustrated position. The loop can be dragged in various directions from this initial position. In each of the following explain your answers.

a) Suppose that the loop is dragged vertically. Just after it starts to move, is there a current in the loop? If so, what is its direction?

b) Suppose that the loop is dragged to the left. Just after it starts to move, is there a current in the loop? If so, what is its direction?

c) Suppose that the loop is dragged to the right. Just after it starts to move, is there a current in the loop? If so, what is its direction?

93 Induced currents



A conducting loop is in the vicinity of a long straight wire. The wire carries a current pointing in the illustrated direction. In each of the following situations describe whether there is a current in the loop and, if so, what it's direction is. Explain your answers.

a) The loop is fixed and the current in the straight wire steadily increases.

b) The loop is fixed and the current in the straight wire steadily decreases.

c) The current in the wire stays constant and the loop is pushed to the right.

d) The current in the wire stays constant and the loop is pushed to the left.

e) The current in the wire stays constant and the loop is pushed up (the page).

94 Induced EMF

A circular loop with radius 10 cm lies in the xy plane. An electromagnetic produces a uniform magnetic field

$$\vec{\mathbf{B}} = 0.0030t^2\,\hat{\mathbf{j}} + 0.0040t^3\,\hat{\mathbf{k}}$$

where the constants have units such that the field is in Teslas.

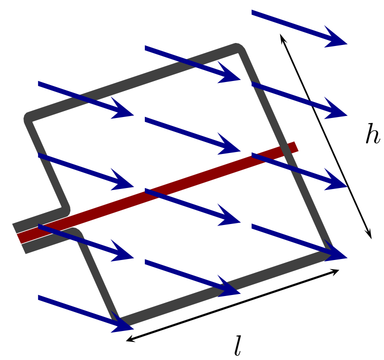
a) Determine an expression for the EMF in the loop at any time.

b) Determine the EMF in the loop at $t = 0.0\,\mathrm{s}$.

c) Determine the EMF in the loop at $t = 10.0\,\mathrm{s}$.

95 Generator

A generator consists of a loop which is forced to rotate about a fixed axle while it is in a uniform magnetic field as illustrated. Suppose that initially the loop is oriented vertically and then is forced (by a hand or something else external) to rotate with constant angular velocity, ω, about the illustrated axle. The aim of this exercise is to determine an expression for the EMF produced around the loop.



a) Sketch a side view of the loop, indicate the normal and the angle between the normal and the field, θ. Express θ in terms of ω and time.

b) Determine an expression for the flux through the loop as a function of time.

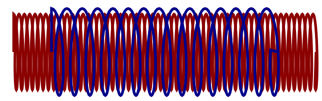
c) Determine an expression for the EMF around the loop as a function of time.

d) Does the EMF remain constant?

e) Suppose that the loop is connected to a resistor with resistance R. Determine an expression for the power delivered to the resistor.

96 Transformer

A simple model of a transformer consists of two solenoids, whose axes lie along the same straight line. Suppose that the inner solenoid has N_1 turns and the outer solenoid has N_2 turns. The two solenoids are arranged so that their radii are essentially the same.



a) Consider any single loop of each solenoid. How does the flux through a single loop of the inner solenoid compare to that through a single loop of the outer solenoid.

b) Let Φ_1 be the total flux through the inner solenoid and Φ_2 be the total flux through the outer solenoid. Find an expression relating these and N_1 and N_2.

c) Let $\mathcal{E}_1$ be the EMF across the inner solenoid and $\mathcal{E}_2$ be the EMF across the outer solenoid. Find an expression relating these and N_1 and N_2.

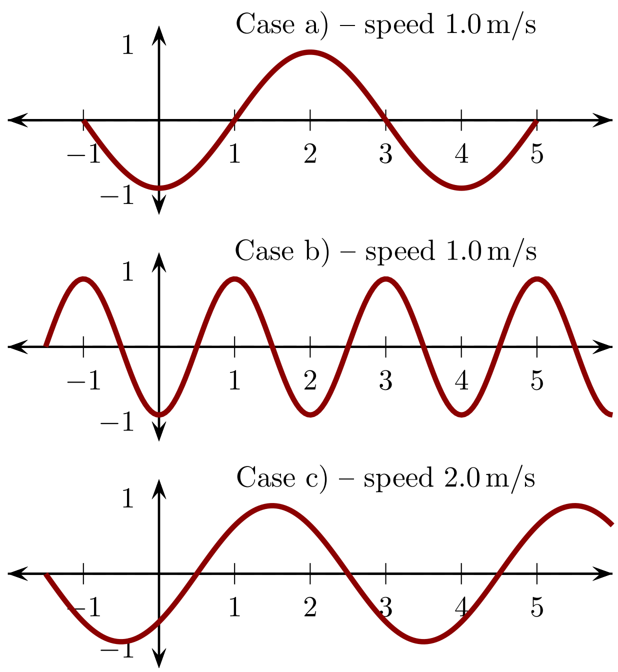
Waves

97 Wavelength and frequency

Various waves on strings are as illustrated. The wave speeds are provided for each case. The units of the axes are meters.

a) Rank the waves in order of increasing wavelength. Indicate equality whenever it occurs. Explain your answer.

b) Rank the waves in order of increasing frequency. Indicate equality whenever it occurs. Explain your answer.

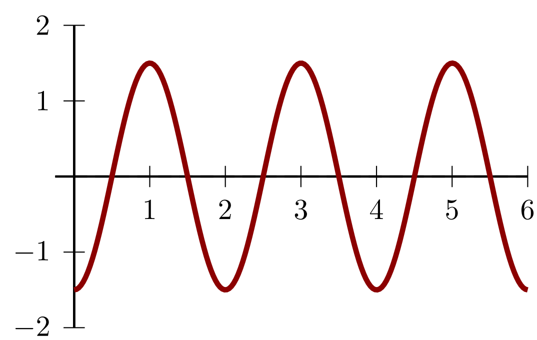


98 Waves on a string

A snapshot of a wave on a string is illustrated. The units of the axes are meters.

a) Determine the wavelength of the wave.

b) The wave is observed as time passes and it is found that 20 crests pass the 4 m mark in 5.0 s. Determine the wave speed.



99 Waves on a string with an oscillating end

The PhET animation "Waves on a String" allows you to visualize transverse waves on a string. Find the animation at

http://phet.colorado.edu/en/simulation/wave-on-a-string

and open it. Adjust the settings as follows:

1. Check the button "No end" at the upper right.

2. Check the button "Oscillate" at the upper left.
3. In the control panel at the bottom, adjust "Damping" to none.
4. In the control panel at the bottom, adjust "Tension" to low.
5. In the control panel at the bottom, check the "Rulers" and "Timer" buttons.

a) Set the frequency to 0.30 Hz. Using the rulers and timer, determine the speed of the wave. Using the ruler, determine the wavelength of the wave. Determine the frequency of the wave by counting crests that pass a given point.

b) Set the frequency to 0.60 Hz. Repeat part a).

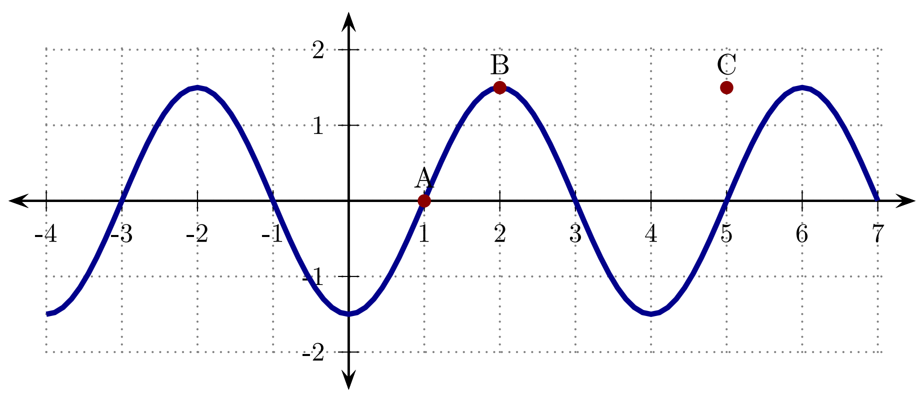
c) Is the speed the same in both cases?

d) Check that $v = \lambda f$ in both cases.

e) For one of the frequencies, vary the amplitude and show that it does not affect the wave speed or wavelength.

100 Sinusoidal waves

A snapshot of a segment of a wave on a string at a particular instant is illustrated. The distances are measured in meters.



a) The portion of the wave at A takes 0.0050 s to complete one cycle. Determine the frequency of the wave.

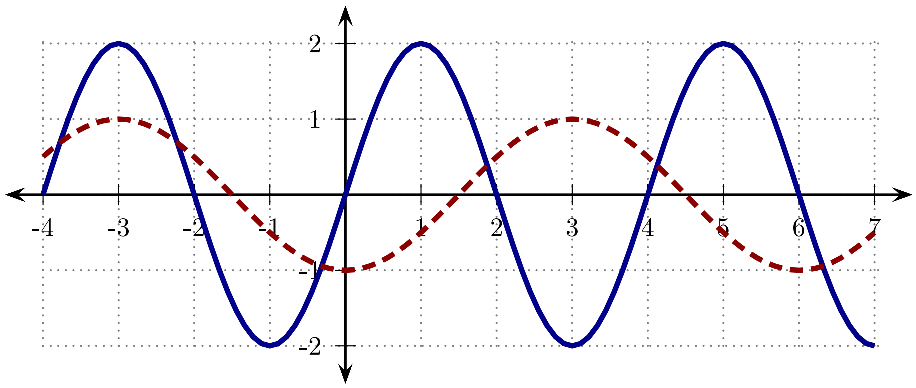
b) Determine the wavelength and speed of the wave.

c) Determine how long it will take the crest labeled B to reach the point C.

d) Using the graph above, sketch the wave at an instant 0.0025 s after that illustrated above.

101 Wave displacement at one instant

A snapshot of two waves on a string at a particular instant ($t = 0$) are illustrated. The distances are measured in meters.



a) Determine an expression of the form $y = A\sin(kx)$, for the wave illustrated by the solid line (i.e. find numbers for A and k).

b) Determine an expression of the form $y = A\sin(kx + \phi)$, for the wave illustrated by the dashed line (i.e. find numbers for A, k and ϕ).

102 Wave displacement function

A wave travels along a string. At one instant a snapshot is taken. The resulting pattern is described by the displacement function

$$y = A \sin(kx).$$

The definition of wavelength means that the displacement at x_0 must be the same as at $x_0 + \lambda$ regardless the value of x_0. Using the expression above, show that if $k = 2\pi/\lambda$, then y evaluated at x_0 must be the same as y evaluated at $x_0 + \lambda$ (regardless of x_0).

103 Wave displacement

a) The displacement for a wave on a string is

$$y = 4.0 \sin(20x - 100t).$$

Determine the amplitude, wavelength, frequency and wavespeed of the associated wave.

b) The general displacement for a sinusoidal wave is

$$y = A \sin(kx - \omega t).$$

Consider this at $t = 0$. At this instant determine expressions for all locations where the displacement is a (positive) maximum. Use this to determine an expression, in terms of k, between any successive maxima.

104 Wave speeds

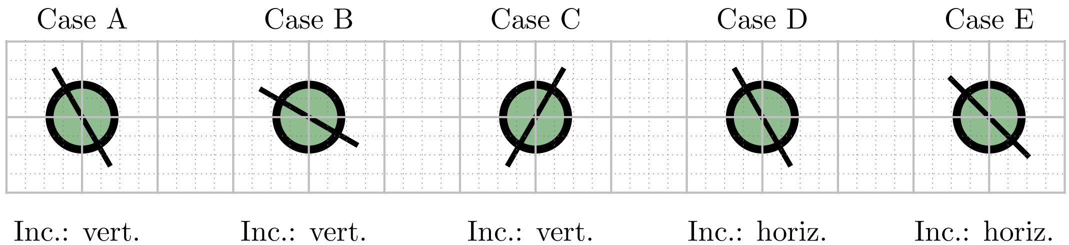
Waves on different strings (A, B, C and D) are described by displacement functions

$$\begin{aligned} y_A &= 5 \sin(4x - 20t), \\ y_B &= 5 \sin(20x - 4t), \\ y_C &= 10 \sin(8x - 40t), \text{ and} \\ y_D &= 10 \sin(4x - 8t) \end{aligned}$$

Rank these in order of increasing wavespeed, indicating equality whenever it occurs. Explain your answer.

105 Polarization filters

Linearly polarized light is incident on polarizing filters whose transmission axes are oriented as illustrated. The diagram describes the polarization axis of the incident light in each case. Rank the intensities of the transmitted light. Explain your answers.



106 Two polarization filters

Unpolarized light, with intensity I_0 is incident on a polarization filter whose transmission axis is vertical. After passing through this filter it is incident on a second filter. Determine the angle from the horizontal at which the transmission axis of the second polarizer must be so that the intensity of the light transmitted by the second filter is $\frac{3}{8}I_0$?

107 Polarization filters

Light that is vertically polarized is incident on a succession of polarization filters. The angle between the transmission axes of any two successive filters is the same and the transmission axis of the final filter is horizontal. Let I_0 be the intensity of the light incident on the first filter and I_f be the light transmitted by the final filter.

a) Suppose that there are three filters. Thus the angles between successive transmission axes is 30°. Determine the ratio I_f/I_0.

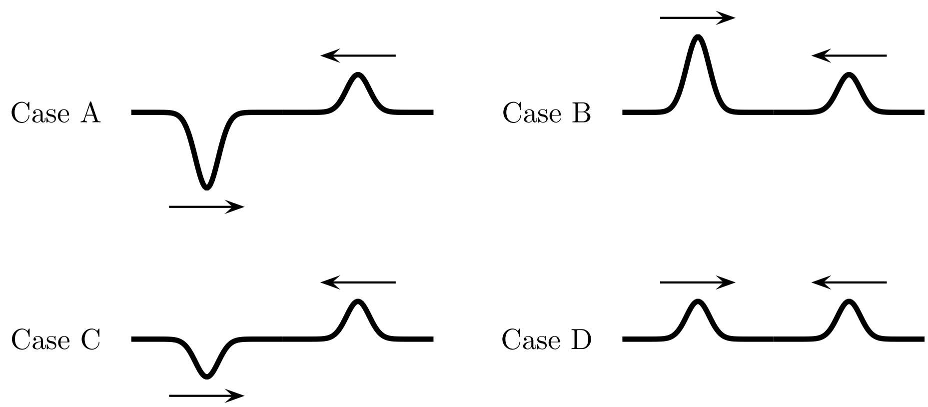
b) Suppose that there are six filters. Thus the angles between successive transmission axes is 15°. Determine the ratio I_f/I_0.

c) Suppose that there are 90 filters. Thus the angles between successive transmission axes is 1°. Determine the ratio I_f/I_0.

d) As more filters are added in this way, what does the ratio I_f/I_0 approach?

108 Interference of pulses

Various pulses approach each other as illustrated.



The pulses overlap and interfere. Rank the situations in order of increasing peak height during interference (indicate any ties in the ranking). Explain your answer.

109 Interference of waves

a) Two sources produce waves that travel (along one direction) with speed 32 m/s. The sources oscillate with frequency 8.0 Hz. By what distances must the sources be separated to produce constructive interference? By what distances must the sources be separated to produce destructive interference?

b) Two sources produce waves that travel in a medium with speed 300 m/s. It is found that the closest separation between the sources that results in destructive interference is 0.050 m. Determine the wavelength and frequency of the waves.

Wave Optics

110 Double slit interference

Light with wavelength λ is incident on a double slit with spacing d. A pattern is observed on a screen which is far from the slits (compared to the separation between the slits).

a) Explain whether each of the following is correct or not.

  i) The pattern looks like a wave with bright fringes separated by a distance λ.

  ii) The pattern looks like a wave with bright fringes separated by a distance d.

  iii) The pattern looks like a wave with the locations of the bright fringes determined by the wavelength only.

  iv) The pattern looks like a wave with the locations of the bright fringes determined by the slit separation only.

b) The wavelength is doubled to 2λ and the slit spacing is also doubled to $2d$. Which of the following is true? Explain your answer.

  i) The separation between the fringes is a quarter of what it was before.

  ii) The separation between the fringes is a half of what it was before.

  iii) The separation between the fringes is the same as before.

  iv) The separation between the fringes doubles.

  v) The separation between the fringes increases by four times.

111 Double slit interference pattern

a) Light with wavelength 650 nm illuminates a double slit where the spacing between the slits is 3500 nm. Determine the angles at which all the bright fringes appear.

b) Light with wavelength λ illuminates a double slit with slit spacing 8.1λ. Determine the number of bright fringes that appear.

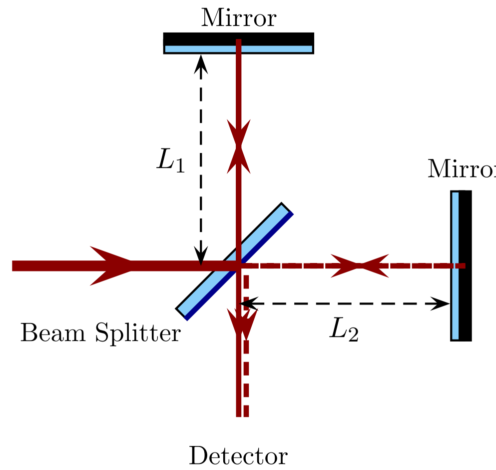
112 Hydrogen emission spectrum

Light from hydrogen is passed through a diffraction grating. It is observed that the spectrum consists of three colors (red, turquoiose, dark blue). It is observed that the first-order bright red fringe occurs at an angle of $7.53°$; the wavelength of this light is 656 nm. Then the first-order turquoise fringe is observed at $5.54°$ while the first-order dark blue fringe is observed at $4.96°$.

a) Determine the wavelengths of the turquoise and dark blue fringes.

b) There is actually a fourth color that is too faint to be observed accurately but whose wavelength is 397 nm. Determine the angles at which the first-order fringe for this would occur.

113 Michelson interferometer



A Michelson interferometer is as illustrated. Initially the light used in the interferometer is produced by a HeNe laser with wavelength 632 nm. When the mirrors are adjusted so that $L_1 = L_2$ it is observed that the intensity of the light at the detector is a maximum.

a) Suppose that initially $L_1 = L_2$. What is the smallest distance that the moveable mirror must be shifted so that the light at the detector appears dark?

b) Suppose that initially $L_1 = L_2$. What is the smallest distance that the moveable mirror must be shifted so that the light at the detector again appears bright?

c) The light at the detector is initially bright. By how far must the moveable mirror be shifted so that the light at the detector cycles through 50 bright fringes?

d) Suppose that light is incident on the interferometer and when the moveable mirror is moved by 0.011 mm the light at the detector cycles through 40 bright fringes. Determine the wavelength of the light.

e) The LIGO observatory uses a Michelson inteferometer to detect gravitational waves. Such a wave would stretch the length of one "arm" by $\Delta L \approx 10^{-22} L$ where L is the length of the "arm." By repeatedly reflecting light back and forth down the arm the apparatus attains an arm length of 1000 km. Determine ΔL when a gravitational wave passes LIGO. Does it seem plausible to measure such a change in position?

f) Analysis of the interference of the two waves predicts that the intensity of the light at the detector is

$$I = I_0 \cos^2 \left(\frac{2\pi \Delta L}{\lambda} \right).$$

Let $\Delta I = I_0 - I$ be the change in intensity as the gravitational wave passes. Determine the fractional change in intensity at the detector $\Delta I / I_0$. For small angles $\cos\theta \approx 1 - \theta^2/2$. Assuming $\lambda = 10^{-6}$ m determine $\Delta I / I_0$. Does it seem plausible to measure such a change?

114 Single slit

Laser light is incident on a single slit whose width can be varied. The diffraction pattern is projected onto a distant screen. The slit with is initially a_0 and is then narrowed to $\frac{a_0}{5}$. Which of the following is true regarding the width of the central maximum on the distant screen? Explain your answer.

i) Width reduces to 1/5 of original width.

ii) Width is halved.

iii) Stays the same.

iv) Width doubles.

v) Width is increases by a factor of 5.

115 Single slit and white light

White light contains a mixture of light of all colors. Suppose that white light is incident on a single slit with width 0.25 mm.

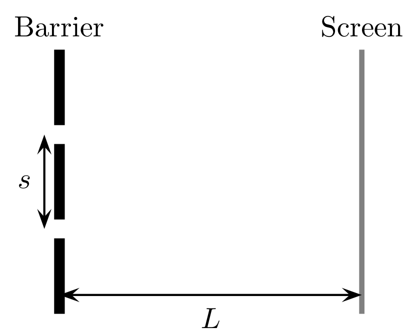
a) Determine the angle at which the first minimum in the diffraction pattern occurs for red light with wavelength 700 nm.

b) Determine the angle at which the first minimum in the diffraction pattern occurs for violet light with wavelength 380 nm.

c) Describe the appearance in terms of colors of the central bright spot in the diffraction pattern.

116 Diffraction from two circular openings

Light with wavelength λ is incident on a barrier that contains two circular openings, each with diameter D. The separation between the centers of the two openings is s.



a) Sketch the two central diffraction maxima produced by the two apertures and use this to provide the mimimum value of s in terms of the width of each maximum, w, such that the two maxima *do not overlap.*

b) Use this result to provide a minimum value of s in terms of λ, D and L such that the two maxima *do not overlap.*

c) Red light with wavelength 650 nm is incident on a piece with two small holes which each have diameter 1.0 mm. Determine the closest spacing between the holes so that they can be distinguished at a distance of 5.0 m.

117 Michelson interferometer

Light with wavelength λ is incident on a Michelson interferometer and the resulting pattern at the detector displays a dark spot. The single moveable mirror is then moved so that the pattern cycles exactly once from a dark spot to a bright spot. Which of the following is the distance moved by the mirror? Explain your answer.

i) λ

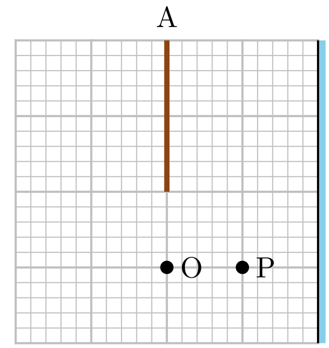
ii) $\frac{3\lambda}{4}$

iii) $\frac{\lambda}{2}$

iv) $\frac{\lambda}{4}$

Geometric Optics

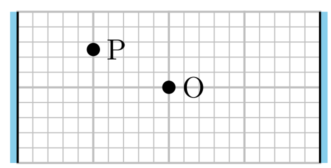
118 Visible image in a mirror



A stick labeled A is held parallel to a flat mirror as illustrated. Two small observers are located at the points labeled O and P. In the following, explain your answers.

a) Suppose that the upper half of the mirror surface is covered by an opaque material. How much of the image of the stick will each observer be able to see?
b) Suppose that the lower half of the mirror surface is covered by an opaque material. How much of the image of the stick will each observer be able to see?
c) Suppose that the lower half of the mirror surface is covered by an opaque material and O moves away from the mirror (in a direction perpendicular to the mirror). As O moves away from the mirror does more or less of the image become visible?

119 Image formation by parallel mirrors

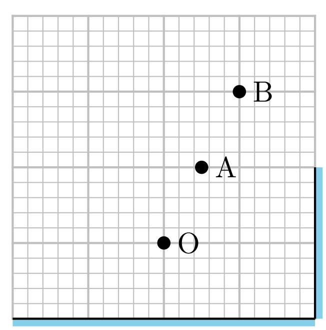


Two mirrors are parallel as illustrated. The distance between them is 4.0 m. An object O is placed midway between the mirrors. Another object is placed at a quarter of the distance between the two mirrors.

a) Determine the distance between successive images of O on either side.
b) Are images of P on the right evenly spaced? Explain your answer.
c) Are images of P on the left evenly spaced? Explain your answer.

If you can determine a formula for the locations of all the images for point P that will be impressive!

120 Image formation by multiple mirrors

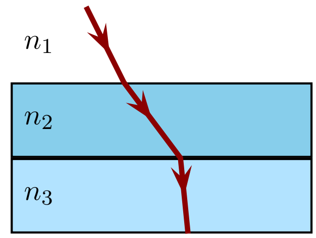


Two mirrors are oriented at right angles as illustrated. An object O is placed at the indicated point. Two observers, A and B are located as illustrated.

a) Recreate the diagram accurately and indicate the locations of all the images of the object O. Explain your answer.
b) Which of the images can observer A see? Explain your answer.
c) Which of the images can observer B see? Explain your answer.

121 Refraction through multiple materials

Two rectangular materials are stacked as illustrated. The indices of refraction of the materials and the surrounding medium are as illustrated. A light beam follows the indicated path. Which of the following is true regarding the indices of refraction? Explain your answer.



i) $n_3 < n_1 < n_2$

ii) $n_1 < n_3 < n_2$

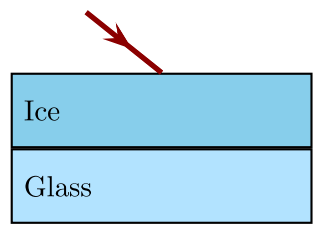
iii) $n_1 < n_2 < n_3$

iv) $n_3 < n_2 < n_1$

v) $n_2 < n_1 < n_3$

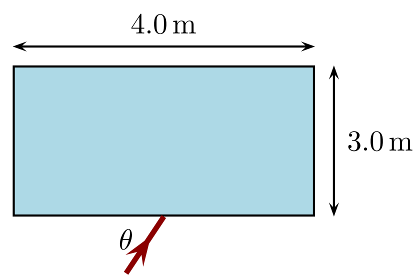
122 Refraction through multiple materials

A rectangular slab of ice, with index of refraction 1.33 lies on top of a rectangular block of glass with index of refraction 1.55. Light traveling through the air is incident in the ice at an angle of 50° from the normal. Determine the angle at which the light travels through the glass.



123 Refraction at multiple surfaces

A rectangular block of glass with index of refraction 1.55 is placed in air. The block has height 1.0 m and length 4.0 m. A light ray is incident midway along the length of the block at an angle θ with respect to the surface.

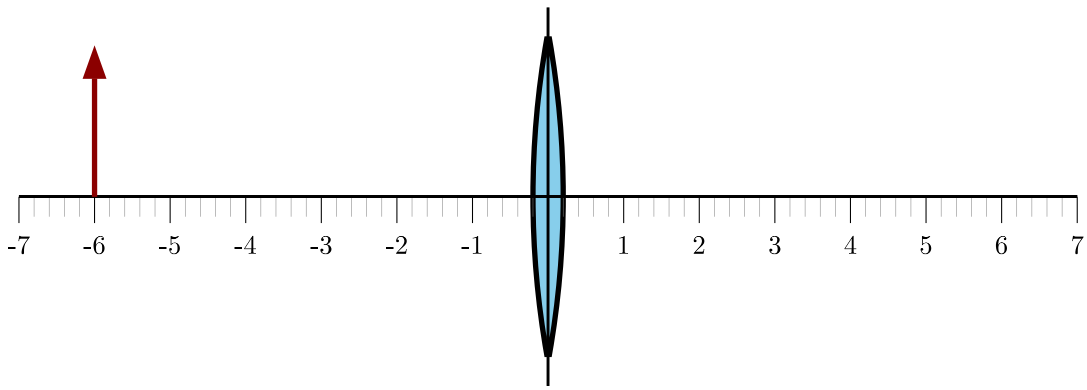


a) Determine where the light emerges back into the air and the direction in which it emerges the block if $\theta = 70°$.

b) Determine where the light emerges back into the air and the direction in which it emerges the block if $\theta = 10°$.

124 Image formation by a convex lens: object beyond focal point

A convex lens has focal length 2.0 cm. An arrow with height 2.0 cm is placed 6.0 cm left of the lens.



a) Trace two rays from the tip of the arrow to determine where the image of the tip is produced.

b) Determine the distance from the lens plane to the image of the arrow.

c) Determine the height of the image of the arrow. Determine the magnification

$$m := \frac{h'}{h}$$

where h is the height of the object and h' is the height of the image.

The thin lens equation relates the positions of the object and the image via

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

where s is the distance from the lens to the object and s' is the distance from the lens to the image.

d) Use the thin lens equation to predict the location of the image. Check this against your diagram.

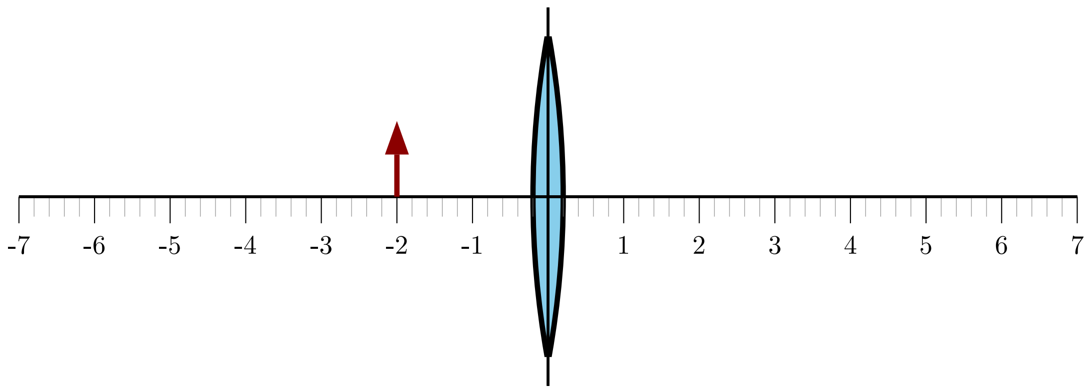
e) The magnification equation predicts

$$m = -\frac{s'}{s}.$$

Use this to predict the magnification and the height of the image. Check this against your diagram.

125 Image formation by a convex lens: object between lens and focal point

A convex lens has focal length 6.0 cm. An arrow with height 1.0 cm is placed 2.0 cm left of the lens.



a) Trace two rays from the tip of the arrow to determine where the image of the tip is produced.

b) Determine the distance from the lens plane to the image of the arrow.

c) Determine the height of the image of the arrow. Determine the magnification

$$m := \frac{h'}{h}$$

where h is the height of the object and h' is the height of the image.

The thin lens equation relates the positions of the object and the image via

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

where s is the distance from the lens to the object and s' is the distance from the lens to the image.

d) Use the thin lens equation to predict the location of the image. Check this against your diagram.

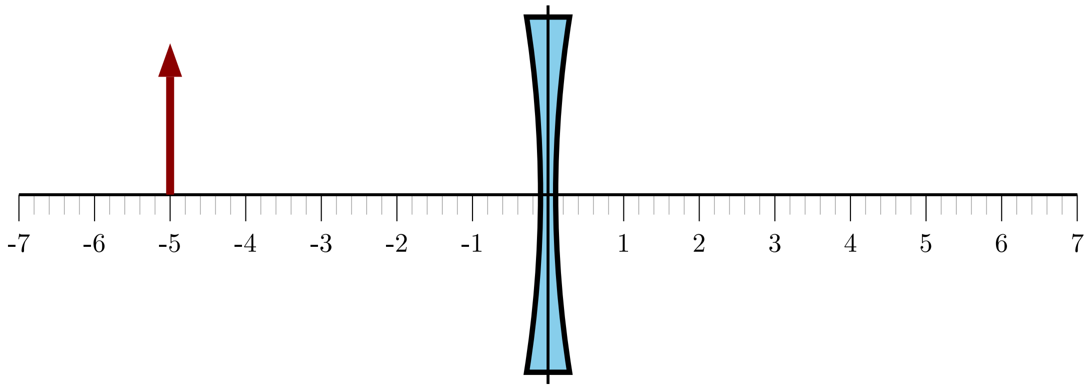
e) The magnification equation predicts

$$m = -\frac{s'}{s}.$$

Use this to predict the magnification and the height of the image. Check this against your diagram.

126 Image formation by a concave lens

A concave lens has focal length $-3.0\,\text{cm}$. An arrow with height $2.0\,\text{cm}$ is placed $5.0\,\text{cm}$ left of the lens.



a) Trace two rays from the tip of the arrow to determine where the image of the tip is produced.

b) Determine the distance from the lens plane to the image of the arrow.

c) Determine the height of the image of the arrow. Determine the magnification

$$m := \frac{h'}{h}$$

where h is the height of the object and h' is the height of the image.

The thin lens equation relates the positions of the object and the image via

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

where s is the distance from the lens to the object and s' is the distance from the lens to the image.

d) Use the thin lens equation to predict the location of the image. Check this against your diagram.

e) The magnification equation predicts

$$m = -\frac{s'}{s}.$$

Use this to predict the magnification and the height of the image. Check this against your diagram.

127 Images formed by a convex lens

An object is placed to the left of a convex lens. The object is between the lens and the focal point.

a) The object is moved closer to the lens. Which of the following is true? Explain your choice.
   i) The image stays in the same place as the object is moved.
   ii) The image moves towards the lens as the object is moved.
   iii) The image moves away from the lens as the object is moved.
   iv) The image is always at the focal point.

b) The object is moved closer to the lens. Which of the following is true? Explain your choice.
   i) The image height increases as the object is moved.
   ii) The image height decreases as the object is moved.
   iii) The image stays constant as the object is moved.

c) Explain whether it is possible for the image to be located between the object and the lens.

d) Explain whether it is possible for the image to be larger than the object (while the object is between the lens and the focal point).

128 Images formed by a concave lens

An object is placed to the left of a concave lens.

a) The object is moved closer to the lens. Which of the following is true? Explain your choice.
   i) The image stays in the same place as the object is moved.
   ii) The image moves towards the lens as the object is moved.
   iii) The image moves away from the lens as the object is moved.
   iv) The image is always at the focal point.

b) The object is moved closer to the lens. Which of the following is true? Explain your choice.
   i) The image height increases as the object is moved.
   ii) The image height decreases as the object is moved.
   iii) The image stays constant as the object is moved.

c) Explain whether it is possible for the image to be located left of the object.

d) Explain whether it is possible for the image to be larger than the object (while the object is between the lens and the focal point).

129 Image formation by a convex lens

An object with height 2.0 cm is placed 5.0 cm to the left of a convex lens with focal length 3.0 cm.

a) Determine the location and height of the image using an accurate ray-tracing diagram.
b) Determine the exact location and height of the image by using equations.

130 Image formation by a convex lens

An object with height 2.0 cm is placed 1.0 cm to the left of a convex lens with focal length 3.0 cm.

a) Determine the location and height of the image using an accurate ray-tracing diagram.
b) Determine the exact location and height of the image by using equations.

131 Image formation by a concave lens

An object with height 2.0 cm is placed 6.0 cm to the left of a concave lens with focal length −8.0 cm.

a) Determine the location and height of the image using an accurate ray-tracing diagram.
b) Determine the exact location and height of the image by using equations.

132 Magnified images produced by convex lenses

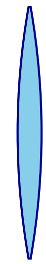
a) An object is placed left of a convex lens with focal length f. Determine, in terms of f, where the object must placed in order to produce an image with magnification -1.
b) An object is placed left of a concave lens with focal length f. Determine, in terms of f, where the object must placed in order to produce an image with magnification $\frac{1}{2}$.

133 Projector

A projector uses a convex lens with focal length 15 cm to form an image of an object, that is inside the projector behind the lens, on a distant screen. The distance between the object and image is fixed at 2.2 m. The lens can be shifted back and forth to produce a clear image.

a) Determine the distance between the object and lens so that a clear image is produced on the screen.
b) Determine the magnification of the image.

134 Focal length of a bi-convex lens



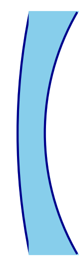
Consider the illustrated lens which is made of glass, for which the index of refraction is 1.50. The lens is surrounded by air.

a) Suppose that the magnitude of the radius of curvature of the left side is 30 cm and of the right side it is 60 cm. Determine the focal length of the lens for an object placed to its left.

b) Suppose that the lens is flipped so that the magnitude of the radius of curvature of the left side is 60 cm and of the right side it is 30 cm. Determine the focal length of the lens for an object placed to its left.

c) Suppose that the *magnitude* of the radius of curvature of the left side is R_{left} and the *magnitude* of the radius of curvature of the right side is R_{right}. Does the focal length of the lens depend on whether the object is left or right of the lens? Explain your answer.

135 Focal length of a convex/concave lens



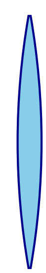
Consider the illustrated lens, for which the index of refraction is 1.50. The lens is surrounded by air.

a) Suppose that the magnitude of the radius of curvature of the left side is 30 cm and of the right side it is 20 cm. Determine the focal length of the lens for an object placed to its left.

b) Suppose that the lens is flipped so that the magnitude of the radius of curvature of the left side is 20 cm and of the right side it is 30 cm. Determine the focal length of the lens for an object placed to its left.

c) Suppose that the *magnitude* of the radius of curvature of the left side is R_{left} and the *magnitude* of the radius of curvature of the right side is R_{right}. Does the focal length of the lens depend on whether the object is left or right of the lens? Explain your answer.

136 Focal length of a lens in different media



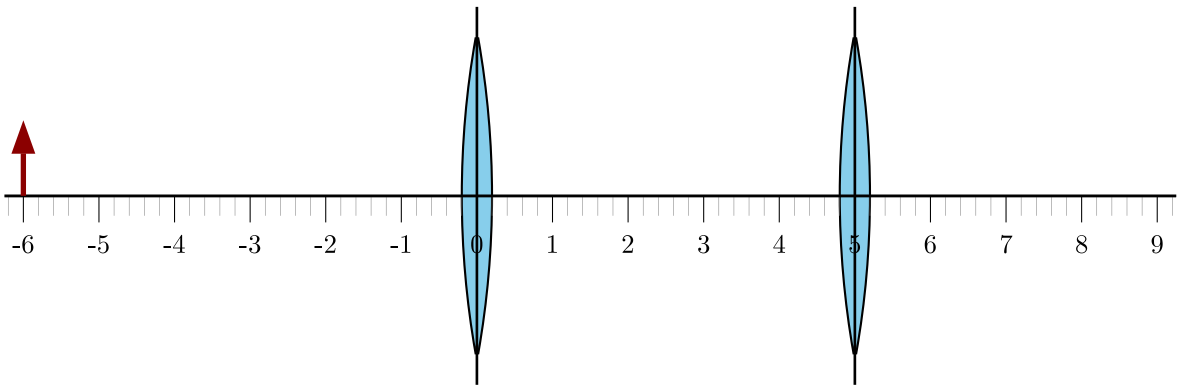
Consider the illustrated lens which is made of glass, for which the index of refraction is 1.50. The radius of curvature of each side is 50 cm.

a) Suppose that the lens is in air (index of refraction 1.00). Determine the focal length of the lens.

b) Suppose that the lens is in water (index of refraction 1.33). Determine the focal length of the lens.

137 Image formation by two convex lenses

Two convex lenses are arranged as illustrated. The lens on the left has focal length 2.0 cm and the lens on the right has focal length 4.0 cm. The lenses are 5.0 cm apart. An arrow with height 1.0 cm is placed 6.0 cm left of the lens on the left.



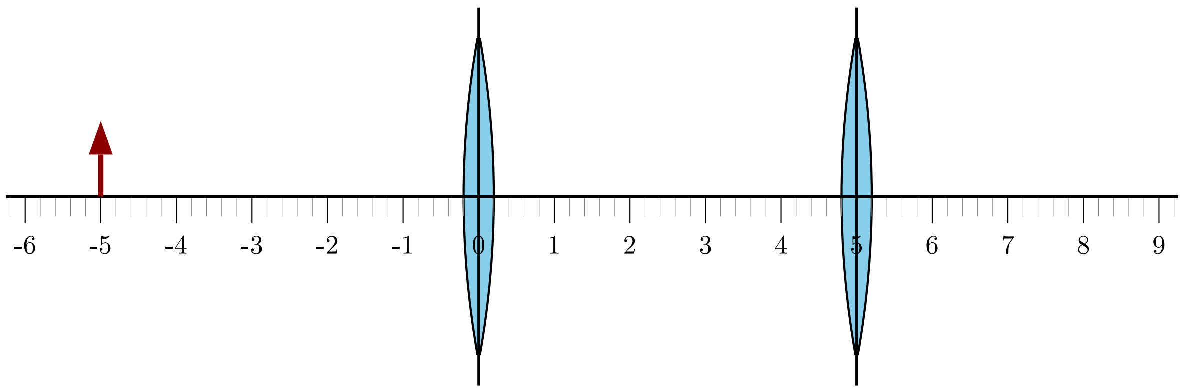
a) Trace two rays from the tip of the arrow to determine the image produced by the lens combination.

b) Use equations to determine the location of the image produced by the lens combination.

c) Use equations to determine the height of the image produced by the lens combination.

138 Image formation by two convex lenses

Two convex lenses are arranged as illustrated. The lens on the left has focal length 1.0 cm and the lens on the right has focal length 1.5 cm. The lenses are 5.0 cm apart. An arrow with height 1.0 cm is placed 5.0 cm left of the lens on the left.



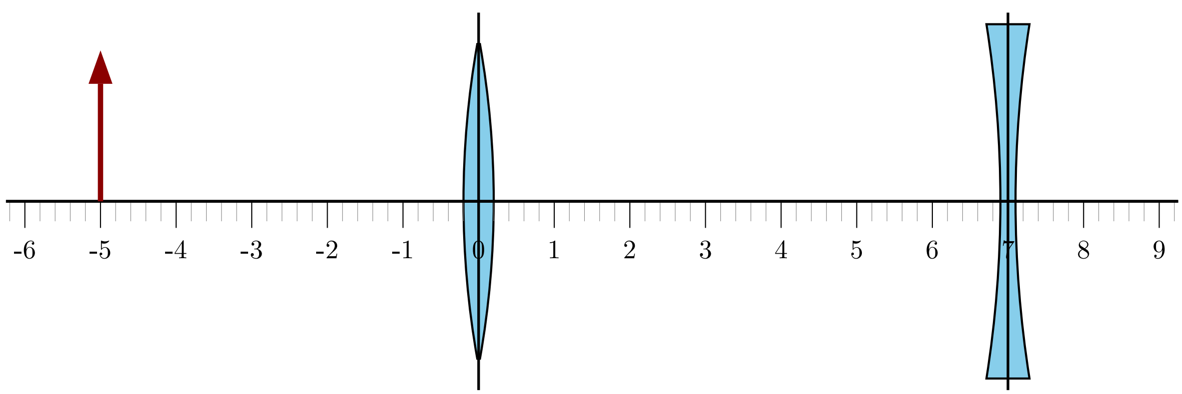
a) Trace two rays from the tip of the arrow to determine the image produced by the lens combination.

b) Use equations to determine the location of the image produced by the lens combination.

c) Use equations to determine the height of the image produced by the lens combination.

139 Image formation by a convex/concave lens combination

Two lenses are arranged as illustrated. The lens on the left has focal length 2.0 cm and the lens on the right has focal length −3.0 cm. The lenses are 7.0 cm apart. An arrow with height 2.0 cm is placed 5.0 cm left of the lens on the left.



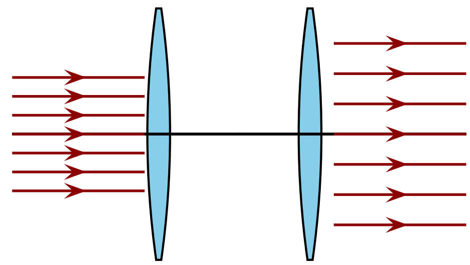
a) Trace two rays from the tip of the arrow to determine the image produced by the lens combination.

b) Use equations to determine the location of the image produced by the lens combination.

c) Use equations to determine the height of the image produced by the lens combination.

140 Beam expander

A beam expander consists of two convex lenses. It is configured so that light rays that enter the expander parallel to the optical axis from the left emerge parallel to the optical axis on the right. Suppose that the focal length of the lens on the left is f_1 and the focal length of the lens on the right is f_2.



a) Determine an expression for the separation of the lenses in terms of f_1 and f_2 so that the incoming and outgoing beams are both parallel to the optical axis.

b) Determine an expression for the factor by which the beam width increases (provided that the incoming and outgoing beams are both parallel to the optical axis).

Standing Waves

141 Standing waves on a string

A 0.80 m string is configured so that waves on the string travel with speed 410 m/s.

a) Sketch the fundamental ($n = 1$) and determine its wavelength and frequency.
b) Sketch the second harmonic ($n = 2$) and determine its wavelength and frequency.
c) Sketch the third harmonic ($n = 3$) and determine its wavelength and frequency.
d) Determine formulas for the wavelengths and frequencies of *all* possible standing waves.

A general expression for the displacement of the string, with length L and both of whose ends are fixed, is

$$y(x,t) = A \sin(kx) \cos(\omega t)$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f$. These satisfy $\omega = kv$.

e) Show that this expression satisfies the requirement for a fixed end at $x = 0$.
f) Since the other end is fixed, at $x = L$, the displacement must be zero. Substitute this into the general expression above and show that it can only be satisfied if k takes certain specific values. Determine what these values are (in terms of L) and use this to obtain expressions for the wavelengths of all possible standing waves.

142 Standing waves on a string

Waves on a particular stretched string travel with speed the string travel with speed 250 m/s. Both ends of the string are held fixed. The fundamental frequency is 512 Hz.

a) Determine the length of the string.
b) Determine the frequency of the second harmonic.
c) Determine the frequency of the third harmonic.
d) Is there a harmonic with a frequency of 1250 Hz. If so which is it?
e) Is there a harmonic with a frequency of 2560 Hz. If so which is it?