

HW due Weds

## Lasers

Lasers produce light via atomic emission processes. Certain quantum mechanical processes endow the emitted light with particular properties such as long coherence times.

Lasers use an amplification process to produce light of large intensity. The process itself involves particular quantum physics processes that we will investigate

## Energy levels in quantum systems.

A typical feature of many quantum systems is that the possible energy states of the system have energy values that form a discrete, rather than continuous spectrum. Consider, as an example, the harmonic oscillator

A classical block + spring oscillator has energy

$$E = K + U_{sp}$$

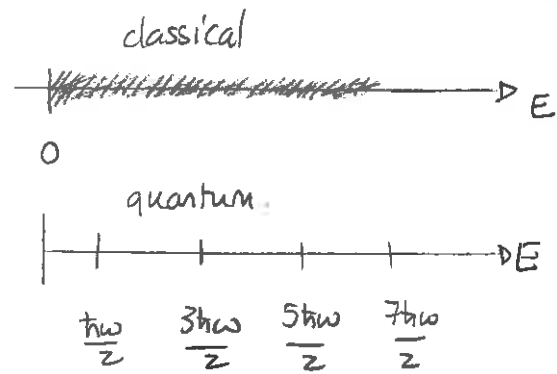
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

and given the various possible initial conditions, the range of possible energies is  $E \geq 0$ . On the other hand, a quantum harmonic oscillator has energies

$$E_n = \hbar\omega \left(n + \frac{1}{2}\right) \quad n = 0, 1, 2, \dots$$

where  $\omega = 2\pi f$  is the angular frequency of oscillation.

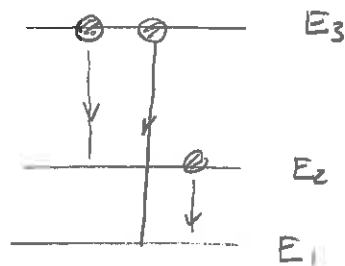
The two can be illustrated graphically showing a discrete energy spectrum for the quantum oscillator and a continuous spectrum for the classical oscillator.



The energy level structure of a quantum system is crucial for understanding the radiation that it can emit or absorb. A true physical description of this entails a complete quantum description of the system and also the electromagnetic field together with a complete specification of the interaction between these. A simplified description is as follows:

Consider a quantum system with various energy levels. Emission of radiation is guided by:

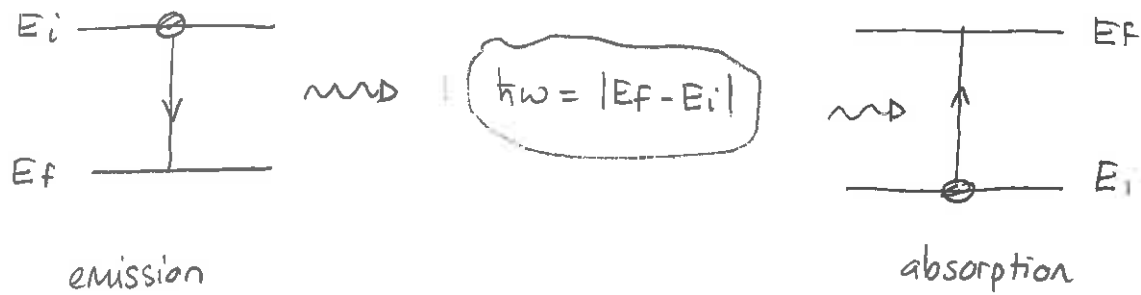
- 1) the system can undergo a transition from a higher energy state to a lower energy state. Possible transitions are illustrated in the example of the diagram.



- 2) when the system undergoes a transition from a higher to lower energy level, the emitted radiation has angular frequency,  $\omega$ , that satisfies:

$$\hbar\omega = |\Delta E_{\text{system}}| \quad \Leftrightarrow \quad \hbar\omega = |E_f - E_i|$$

The same process operates in reverse for absorption. Schematically:



In practice not all transitions are possible. In many quantum systems, there exist selection rules that single out certain transitions as fairly likely and all others as extremely unlikely.

Quantum physics then endeavors to determine energy levels and selection rules for various systems. We review some of these.

### Hydrogen atom

The Schrödinger equation can be solved exactly for the hydrogen atom. The result is that the energy states are indexed by various "quantum numbers:"

- $n = 1, 2, 3, \dots$  principle quantum number (describes energy)
- $l = 0, 1, 2, \dots, n-1$  describes magnitude of orbital angular momentum
- $m_l = -l, -l+1, \dots, l-1, l$  " one component of orbital ang. mom.
- $s = 1/2$  " magnitude of spin angular momentum.
- $m_s = -s, -s+1, \dots, s-1, s$  " one component of spin ang. mom.

The system energy only depends on the principle quantum number and is

$$E_n = - \frac{M_e e^4}{8h^2 \epsilon_0^2} \frac{1}{n^2} = -13.6 \text{ eV} \frac{1}{n^2} \quad n=1,2,3$$

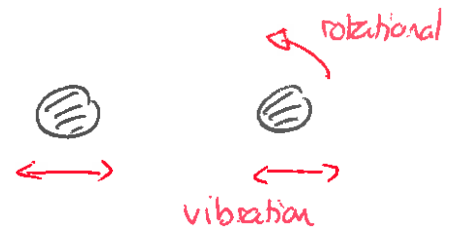
It is then possible to determine the hydrogen atom spectrum by considering transitions such as

$$\begin{aligned} n=2 &\rightarrow n=1 \\ n=3 &\rightarrow n=2 \\ n=3 &\rightarrow n=1 \\ &\vdots \end{aligned}$$

### Molecular systems

Molecules consist of two or more nuclei plus electrons. When considering the system energy, there are several ingredients:

- 1) electronic energy levels - no known analytic expressions
- 2) vibrational energy levels
- 3) rotational energy levels



It is possible to attain expressions for the latter. The vibrational levels are described by an integer  $\nu = 0, 1, 2, \dots$  and energy

$$E_\nu = \hbar \omega_0 \left( \nu + \frac{1}{2} \right) = h f_0 \left( \nu + \frac{1}{2} \right)$$

where  $\omega_0 = 2\pi f_0$  is an angular frequency associated with the vibration

The rotational energy levels are described by

$$E_J = \frac{h^2}{8\pi^2 I} J(J+1) \quad J = 0, 1, 2, 3, \dots$$

where  $I$  is the moment of inertia of the atom. With  $\hbar = h/2\pi$ , we get

$$E_J = \frac{\hbar^2}{2I} J(J+1)$$

Combining these gives the rotational-vibrational energy level values:

$$E_{v,J} = \hbar\omega_0(v + \frac{1}{2}) + \frac{\hbar^2}{2I} J(J+1) \quad \begin{array}{l} J = 0, 1, 2, \\ v = 0, 1, 2, \dots \end{array}$$

There is one important selection rule associated with these:

In any transition  $J$  can only change by  $\pm 1$

In most molecules  $\hbar\omega_0 \gg \frac{\hbar^2}{2I}$  and so the vibrational spectrum has a greater spacing than the rotational spectrum.

Exercise: Ignore the vibrational aspects of a molecule.

- list the four lowest rotational energy levels (in terms of  $\frac{\hbar^2}{2I}$ )
- list the possible frequencies of light emitted or absorbed
- " " " wavelengths " " " "

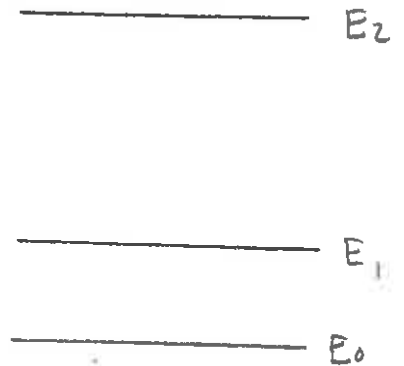
Answer: a)  $E_J = \frac{\hbar^2}{2I} J(J+1)$

$$E_0 = 0 \frac{\hbar^2}{2I}$$

$$E_1 = 2 \frac{\hbar^2}{2I}$$

$$E_2 = 6 \frac{\hbar^2}{2I}$$

$$E_3 = 12 \frac{\hbar^2}{2I}$$



These are not evenly spaced

b) we need  $|\Delta E| = |E_J - E_{J-1}| = \hbar\omega \quad J=1,2,3,-$

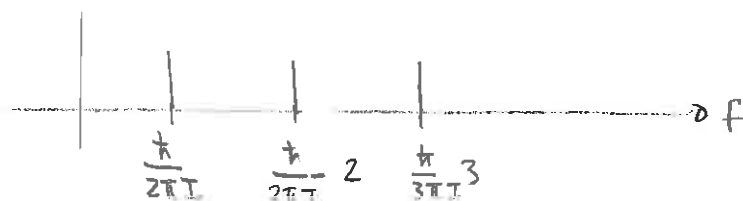
$$\Rightarrow \frac{\hbar^2}{2I} (J(J+1) - (J-1)J) = \hbar\omega$$

$$\Rightarrow \frac{\hbar^2}{2I} \cancel{J} = \hbar\omega$$

$$\Rightarrow \frac{\hbar J}{I} = \omega$$

$$\Rightarrow \frac{\hbar J}{I} = 2\pi f \Rightarrow f = \frac{\hbar}{2\pi I} J \quad J=1,2,3 \dots$$

Thus the spectral lines are evenly spaced.



c) The wavelength of emitted light satisfies

$$\lambda f = c$$

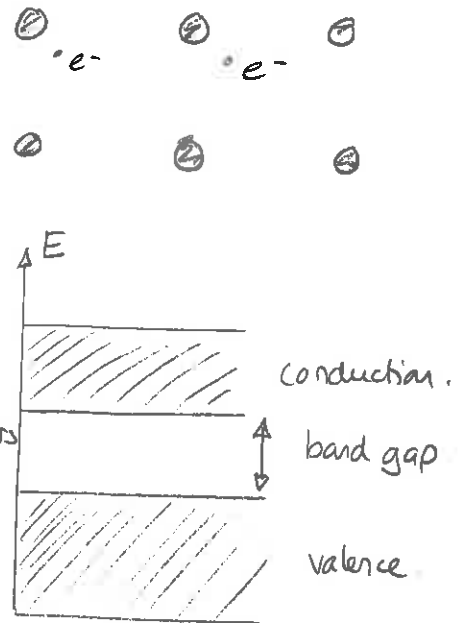
$$\Rightarrow \lambda = \frac{c}{f} \quad \Rightarrow \quad \lambda = \frac{c}{\frac{h J}{2\pi I}}$$

$$\Rightarrow \lambda = \frac{2\pi I c}{h} \frac{1}{J} \quad J=1,2,3$$

and these are not evenly spaced.

### Solid materials

Solids have regular lattices of nuclei. Any electrons are subject to the potential from all nuclei. An analysis of the energy states associated with the electron reveals that allowed energy values occur in continuum ranges separated by energy gaps. This is the band structure of the material. In insulators the lower band or valence band is completely populated and the entire arrangement of electrons is in a static state. In semiconductors thermal excitations can populate the upper energy band. These can emit light when they make the transition to the lower band.



If  $E_g$  is the energy gap then.

$$|\Delta E| \geq E_g$$

means

$$h\nu \geq E_g \Rightarrow hf \geq E_g$$

$$\Rightarrow f \geq E_g/h$$

which implies a minimum absorption or emission frequency. Then

$$f = c/\lambda \Rightarrow \frac{c}{\lambda} \geq E_g/h$$

$$\Rightarrow \frac{hc}{E_g} \geq \lambda$$

which implies a maximum absorption or emission wavelength