Phys 473 2015

Lecture 31

HW Weds

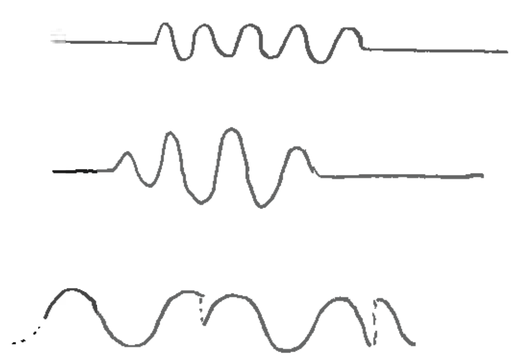
Coherent and incoherent light sources

The preceeding discussions of interferometry have considered the production of interference patterns when the light source is monochromatic, i.e. produces waves of one wavelength or frequency only. We now broaden this to consider source beams that are not purely monochromatic. Snapshots of various possibilities are illustrated

pure monochromatic

"single sinusoidal wave"

non - monochromatic



In the pure sinusoidal case the wave can be described as

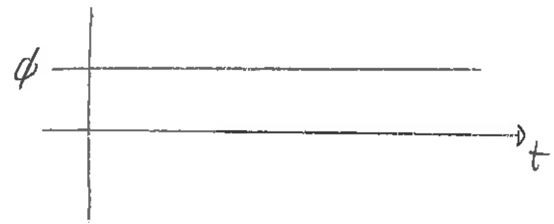
$$E = E_0 \sin(kx - \omega t + \phi) \quad \text{or} \quad E = E_0 e^{i(kx - \omega t + \phi)}$$

where ϕ is a constant. In the other cases it is either a superposition of sinusoidal waves with different k values or it is of the form

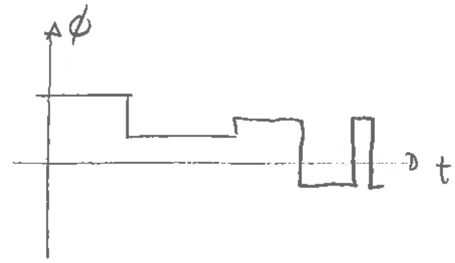
$$E = E_0 e^{i(kx - \omega t + \phi(x,t))}$$

where the phase changes randomly with time.

Plotting the phase would give

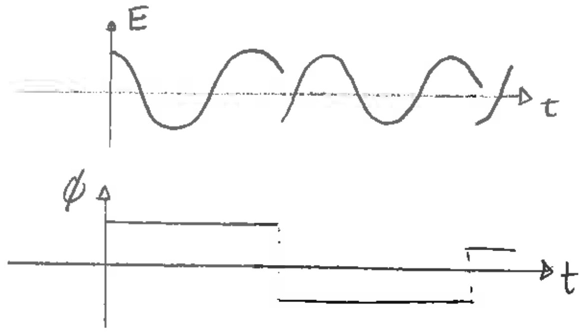


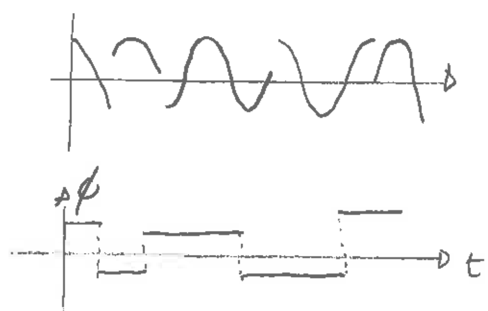
pure monochromatic



non-monochromatic - randomly shifting phase.

Light of the pure monochromatic type where the <u>phase remains constant</u> is called <u>coherent</u>. Clearly all but idealized infinitely long sinusoidal waves are not coherent. The issue then becomes one of how frequently the phase jumps. Two examples are illustrated below





The example on the left has a greater degree of coherence than that on the right. We would like some measure of the coherence of a wave and how this enters into observable phenomena associated with light. The reasons for studying this include:

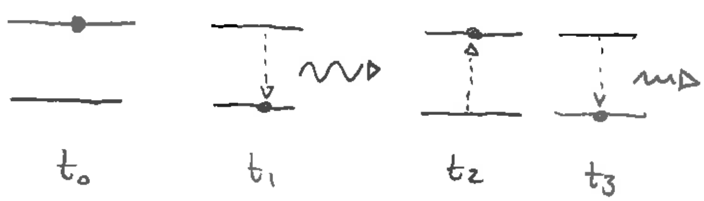
1) describing the effectiveness of a light source in terms of producing interference phenomena

2) relating physical processes within the systems that produce light to the actual light produced.

3) eventually developing measurable tools which decide whether classical and quantum descriptions of light are applicable.

Aside from abstract mathematical developments the quantification of coherence will be closely attached to describing interference phenomena.

The second point can be illustrated by considering two ways in which atomic sources emit light. In general an atom will emit light when it spontaneously decays from an excited to a "ground" state. Suppose that the atom is initially in an excited state. At some time it decays, is later re-excited, etc,...



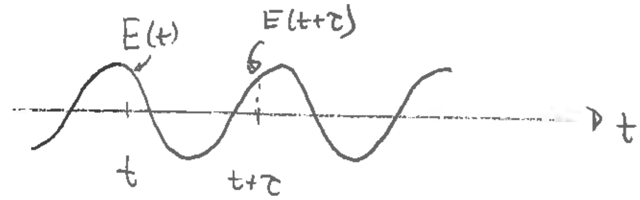
The times between successive decays will be randomly distributed. Thus for such sources, the phase of the emitted waves will vary randomly with these delays between successive decays. But for lasers light is produced by stimulated emission and in these processes the phases between successive emissions are highly correlated. Clearly there is a connection between the physics of emission and the light produced.

The third point requires a quantized theory of the electromagnetic field. For optical frequency fields this leads to the field of quantum optics. This describes the electromagnetic field in terms of various possible quantum states, gives a method for extracting coherence information from these and relates those to intensity measurements. The results are that some states produce coherence information that is identical to that resulting from classical electromagnetism and optics. However other states give coherence results that are provably inexplicable by classical electromagnetism. Such results have been observed experimentally and the conclusion is that certain types of light can only be described via quantum physics.

Coherence.

The way that coherence is measured is to compare the wave at one instant to the wave at other instants.

We pick any time t and determine $E(t)$. We then consider another time $t+\tau$ and determine $E(t+\tau)$.

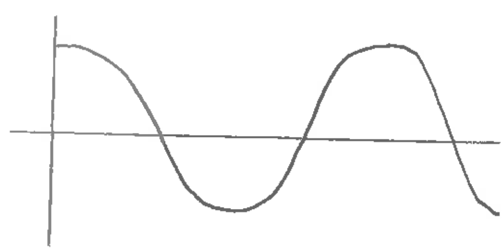


The two will not be equal and the relationship between one and the other will vary as τ varies. But as t varies, if we keep τ fixed then certain patterns emerge for purely sinusoidal waves. For example, consider

$$E(t) = E_0 \cos(\omega t)$$

for some frequency ω and suppose τ is such that

$\omega\tau = 2\pi$.



Then

$$E(t+\tau) = E(t) \quad \text{regardless of } t$$

If τ is such that $\omega\tau = \pi$ then $E(t+\tau) = -E(t)$ regardless of t. In these cases there is a clear correlation between $E(t)$ and $E(t+\tau)$ regardless of t. To quantify this we consider the product

$$E(t)E(t+\tau)$$

over all values of t but for a fixed τ. We average this over all times t giving

$$\lim_{T\to\infty} \frac{1}{T}\int_t^{t+T} E(t')\,E(t'+\tau)\,dt'$$

Exercise: a) Suppose that

$$E = E_0 \cos(\omega t)$$

Determine the average. Does it depend on t. What about τ?

b) Describe how to extend this to a complex representation

$$E_c = E_0 e^{i\omega t}$$

Answer: a) $$E(t')E(t'+\tau) = E_0^2 \cos(\omega t')\cos(\omega t' + \omega\tau)$$

$$\Rightarrow \int_t^{t+T} E(t')E(t'+\tau)\,dt'$$

$$= E_0^2 \int_t^{t+T} \cos(\omega t')\cos(\omega t' + \omega\tau)\,dt'$$

Now $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\Rightarrow \int \cdots = \frac{E_0^2}{2}\left\{\int_t^{t+T} \cos(\omega t' + \omega t' + \omega\tau)\,dt' + \int_t^{t+T} \cos(\omega\tau)\,dt\right\}$$

$$= \frac{E_0^2}{2}\frac{1}{2\omega'}\sin(2\omega t' + \omega\tau)\Big|_t^{t+T} + \frac{E_0^2}{2}T\cos(\omega\tau)$$

Now taking the average gives

$$\frac{E_0^2}{4\omega T}\left\{\sin[\ldots] - \sin[\ldots]\right\} + \frac{E_0^2}{2}T\cos(\omega\tau)$$

Finally taking the limit gives:

$$\lim_{T\to\infty} \frac{1}{T}\int_t^{t+T} E(t')\,E(t'+\tau)\,dt' = \frac{E_0^2}{2}\cos(\omega\tau)$$

This does not depend on t but does depend on τ. Clearly what matters, for a wave of given frequency is only the time interval between the two observation points and not the initial point (at least when averaged over all initial points)

b) Here the real field is

$$E = \frac{1}{2}(E_c + E_c^*)$$

and

$$E(t)\,E(t+\tau) = \frac{1}{2}\left[E_c(t) + E_c^*(t)\right]\left[E_c(t+\tau) + E_c^*(t+\tau)\right]$$

$$= \frac{1}{2} E_0^2 \left\{ e^{(2i\omega t + i\omega\tau)} + e^{i\omega\tau} + e^{-i\omega\tau} + e^{-2i\omega t - i\omega\tau} \right\}$$

Then averaging over t will result in

$$\lim_{T\to\infty} \frac{1}{T}\int E(t')E(t'+\tau)\,dt' = \frac{1}{2}E_0^2\, 2\cos\omega\tau = E_0^2\cos(\omega\tau)$$

We see that this can be attained by computing

$$E_c(t)\,E_c^*(t+\tau) = E_0^2 e^{-i\omega\tau} = \langle E_c(t)\,E_c^*(t+\tau)\rangle$$

and taking the real part

$$\mathrm{Re}\left\{\langle E_c(t)\,E_c^*(t+\tau)\rangle\right\}$$

When working with complex representations we can quantify the coherence via the coherence function or correlation

$$\boxed{\Gamma(\tau) = \langle E(t)\, E^*(t+\tau)\rangle = \lim_{T \to \infty} \int_0^T E(t)\, E^*(t+\tau)\, d\tau.}$$

One case is illustrative. Suppose that

$$E(t) = E_0 e^{i\omega t}$$

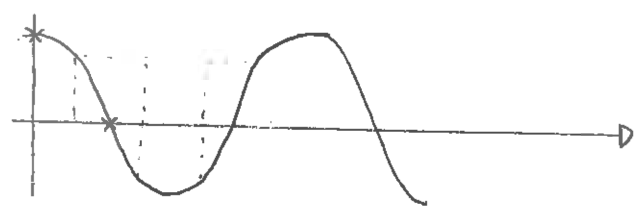
Then

$$\Gamma(t) = E_0^2\, e^{i\omega\tau}$$

and $\mathrm{Re}\{\Gamma(t)\} = E_0 \cos\omega\tau$. Then if $\omega\tau = \pi/2$ we get 0 (but only for special values of τ). Consider the real representation

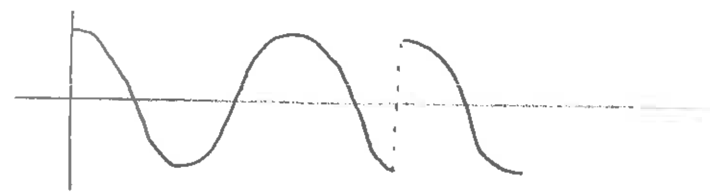
$$E(t) = E_0 \cos(\omega t)$$

ωt	$\omega(t+\tau)$	$\cos(\omega t)\cos(\omega(t+\tau))$
0	$\pi/2$	0
$\pi/4$	$\frac{3\pi}{4}$	$-\frac{1}{2}$
$\pi/2$	π	0
$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{1}{2}$
π	$\frac{3\pi}{2}$	0
$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$-\frac{1}{2}$



We see that the product varies between positive negative and zero as the initial comparison point (t) varies. It will average to zero. But for a pure monochromatic wave this only occurs for special values of τ.

However when the phase jumps randomly, it will be more likely that the average of $E(t)E^*(t+\tau)$ is lowered regardless of τ. For example if $\omega\tau = 2\pi$ the case below indicates that



$E(t)E(t+\tau)$

some times yields positive, sometimes negative values depending on t (unlike always positive or zero depending on t). So the average will be less than $\cos(\omega\tau) = \cos 2\pi = 1$ as before. We can see this using a complex representation with a time dependent phase

$$E = E_0 \, e^{i[\omega t + \phi(t)]}$$

Then

$$E(t)E^*(t+\tau) = E_0^2 \, e^{i[\omega t + \phi(t)]} \, e^{-i[\omega t + \omega\tau + \phi(t+\tau)]}$$

$$= E_0^2 \, e^{i[\phi(t) - \phi(t+\tau)]} \, e^{i\omega\tau}$$

Let $\Delta\phi(t) = \phi(t+\tau) - \phi(t)$. This gives

$$\mathrm{Re}\langle E(t)E^*(t+\tau)\rangle = E_0^2 \langle \cos[\omega\tau - \Delta\phi(t)]\rangle$$

For a fixed phase this gives $E_0^2 \cos(\omega\tau)$ but for randomly varying phase shifts $\Delta\phi(t)$ the cosine averages to zero.