Phys 473 2015

Lecture 29

Interferometry

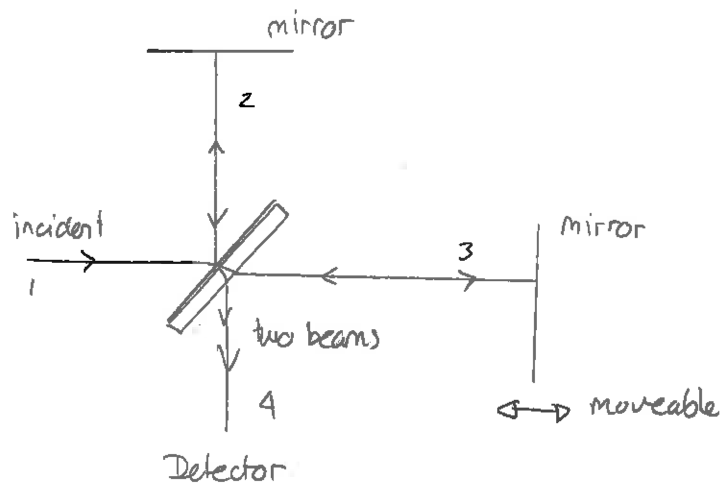
Interferometry is a process by which incident light is split into two beams which are subsequently recombined. The resulting recombined beams interfere, and the interference pattern reveals information about the paths taken by the beams.

An example is a Michelson interferometer which is illustrated. The process is:

1) an incident beam is directed at a beam splitter. The beam splitter partly reflects + partly transmits

2) the reflected ray travels toward a mirror, which is oriented so that it is reflected back

3) the transmitted beam is also reflected to a mirror but this can be moved



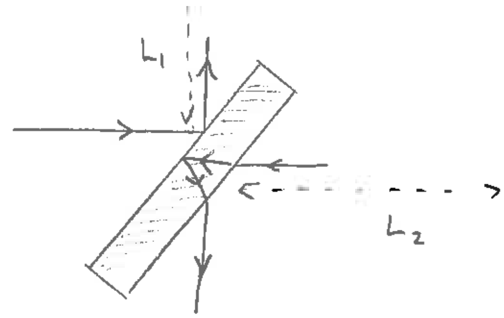
4) A component of each beam can propagate as illustrated. These two beams overlap + interfere. The superposition is detected at a screen

The key aspects of analyzing such an interferometer are:

1) the distances traveled in steps 2 and 3

2) phase shifts on reflection at the beam splitter.

Exercise. Suppose that reflection is off the source side surface of the beam splitter. Assume that the beam splitter has negligible thickness and let L_1 and L_2 be the indicated distances to the mirrors.



a) For what values of $\Delta L = L_2 - L_1$ will monochromatic light of wavelength λ produce maximum intensity at the detector?

b) For what values of ΔL will minimum intensity result?

Answer: a) Constructive interference is required. So the phase shift δ between the waves must be an even multiple of π. So

$$\delta = 2n\pi$$

Now δ = phase shift due to extra distance + phase shift from reflection

$$= \frac{2\Delta L}{\lambda} 2\pi + \pi - 0$$

π ↪ shift for 3 $(n_i > n_t)$

0 ↳ shift for 2 $(n_i < n_t)$

$$\Rightarrow 2n\pi = \frac{4\Delta L}{\lambda}\pi + \pi \Rightarrow (2n-1)\lambda = 4\Delta L$$

$$\Rightarrow \Delta L = \frac{(2n-1)}{4}\lambda \qquad \text{e.g. } \Delta L = \pm\frac{\lambda}{4}, \pm\frac{3\lambda}{4}, \pm\frac{5\lambda}{4}, \ldots$$

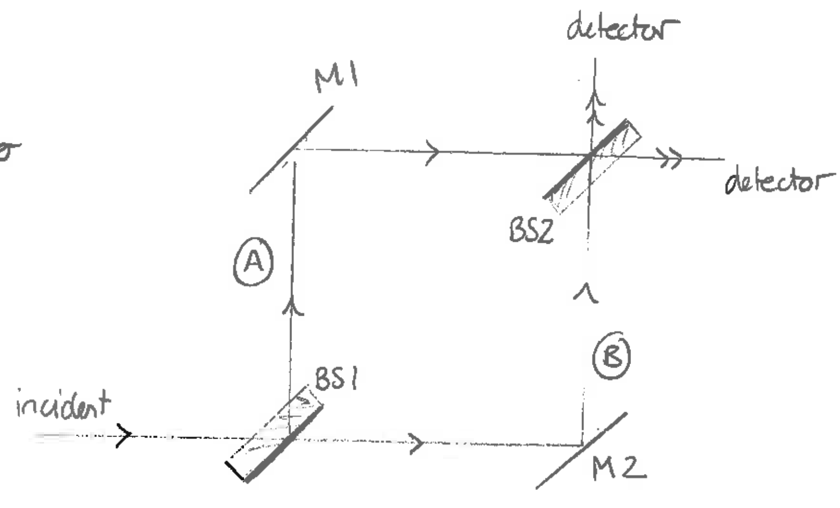
b) $$\delta = (2n+1)\pi \Rightarrow (2n+1)\pi = \left(\frac{4\Delta L}{\lambda} + 1\right)\cancel{\pi}$$

$$\Rightarrow 2n = \frac{4\Delta L}{\lambda} \Rightarrow \Delta L = \frac{n}{2}\lambda$$

$$\Delta L = 0, \pm \lambda/2, \pm 3\lambda$$

Mach - Zehnder Interferometer.

A Mach - Zehnder interferometer uses two beamsplitters and two mirrors as illustrated. The process is:



1) a beam from the source is incident on a beam splitter (BS1).
2) this is split into two beams which travel along separate paths
3) these beams are redirected by mirrors toward another beam splitter (BS2)
4) At BS2, beam A is partly transmitted and partly reflected toward two detectors
5) At BS2, beam B is partly transmitted + partly reflected toward two detectors.
6) There are two beams traveling to each detector. These pairs overlap + interfere.

We aim to determine the electric field and intensity at each detector. We note that each detector records as time passes. So we aim for an electric field as a function of time. There are two factors here:

a) the different path lengths traveled by the beams.
b) the reflection + transmission properties of the beam splitters.

a) Path length issues

The path length effects can be considered by noting that BS 1 produces two effective sources which oscillate which a fixed phase relationship. These then travel linearly to the detector. This is analyzed by considering a simpler situation

IF the sources emit monochromatic waves with wave number k and frequency ω, then the field produced by each source is:

$$E_1(x,t) = E_{10}\, e^{i[k(x-x_1) - \omega t + \varphi_1]}$$

$$E_2(x,t) = E_{20}\, e^{i[k(x-x_2) - \omega t + \varphi_2]}$$

and the combined field is:

$$E = E_1 + E_2 = \left[E_{10}\, e^{i(-kx_1+\varphi_1)} + E_{20}\, e^{i(-kx_2+\varphi_2)}\right] e^{i(kx-\omega t)}$$

$$= E_0\, e^{i(kx-\omega t)}$$

This indicates a field with frequency ω and wavenumber k, and complex amplitude E_0.

Exercise: Show that the E_0 can be written as a complex exponential multiplied by a term that only depends on the phase difference and the path length difference.

Show that the intensity only depends on these differences.

<u>Answer</u> Let $\alpha_1 = (-kx_1 + \varphi_1)$

$\alpha_2 = (-kx_2 + \varphi_2)$

Then $$E_0 = e^{i(\alpha_1+\alpha_2)/2}\left[E_{10}\, e^{i(\alpha_1-\alpha_2)/2} + E_{20}\, e^{-i(\alpha_1-\alpha_2)/2}\right]$$

and $\alpha_1 - \alpha_2 = k(x_2 - x_1) + (\varphi_2 - \varphi_1) = k\Delta x + \Delta\varphi$

The intensity is $$I = \tfrac{1}{2}\epsilon v\, E\, E^* = \tfrac{1}{2}\epsilon v\, E_0 E_0^*$$

and only the term in braces survives giving

$$E_0 E_0^* = \left[E_{10}\, e^{i(k\Delta x + \Delta\varphi)/2} + E_{20}\, e^{-i(k\Delta x + \Delta\varphi)/2}\right]\cdot\left[\ \ldots\ \right]^*$$

$$= |E_{10}|^2 + |E_{20}|^2 + E_{10}E_{20}^*\, e^{i(k\Delta x + \Delta\varphi)} + E_{10}^* E_{20}\, e^{-i(k\Delta x + \Delta\varphi)}$$

If E_{10} and E_{20} are real this gives

$$E_0 E_0^* = E_{10}^2 + E_{20}^2 + 2E_{10}E_{20}\cos[k\Delta x + \Delta\varphi]$$

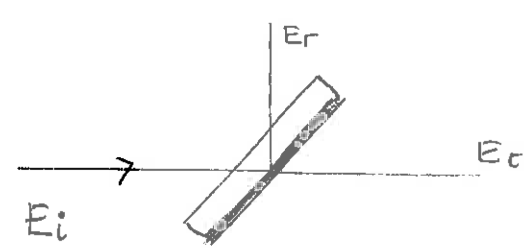
We see that the time terms are irrelevant and only $\Delta k, \Delta\varphi$ matter.

b) <u>Beam splitter</u>

A beam splitter can be described in terms of electric fields.
Then for a beam incident on the non-reflective side

$$E_r = r E_i$$

$$E_t = t E_i$$



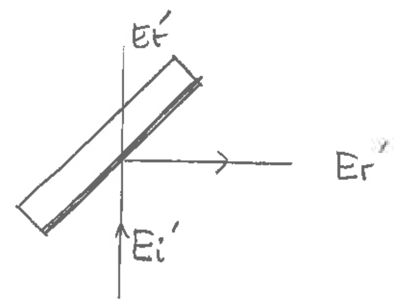
where r and t are complex reflection + transmission coefficients.

So

$$r = |r| e^{i\phi_r}$$
$$t = |t| e^{i\phi_t}$$

and in the illustrated case. $\phi_t = 0$, $\phi_r = \pi$ capture the phases on reflection + transmission. With a beam incident from the other side:

$$E_r' = r' E_i'$$
$$E_t' = t' E_i'$$



with $r' = |r'| e^{i\phi_r'}$

$t' = |t'| e^{i\phi_t'}$

The reflection and transmission coefficients cannot be arbitrary. For a lossless beam splitter energy must be conserved.

Exercise: Use energy conservation, rephrased in terms of intensity, to determine a condition for $|r|$ and $|t|$ when a beam is incident from the left in the indicated diagrams. In a 50-50 beam splitter the energy is split evenly. In this case determine

Answer: Energy conservation implies:

$$I_t + I_r = I_i \quad \Rightarrow \quad \tfrac{1}{2} \epsilon v |E_t|^2 + \tfrac{1}{2} \epsilon v |E_r|^2 = \tfrac{1}{2} \epsilon v |E_i|^2$$

$$\Rightarrow \quad |E_t|^2 + |E_r|^2 = |E_i|^2$$

$$\Rightarrow \quad |t|^2 + |r|^2 = 1$$

For a 50-50 splitter $|t|^2 = 1/2 \Rightarrow |t| = \frac{1}{\sqrt{2}}$

$|r|^2 = 1/2 \Rightarrow |r| = \frac{1}{\sqrt{2}}$

Thus we get

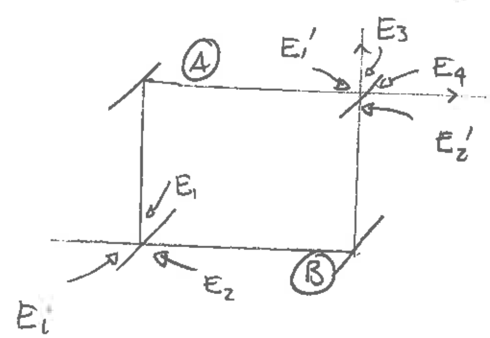
$$\boxed{\begin{aligned}|r|^2 + |t|^2 &= 1 \\ |r'|^2 + |t'|^2 &= 1.\end{aligned}}$$

Separately, we can show that

$$\boxed{\begin{aligned}|r| &= |r'| \\ |t| &= |t'| \\ \phi_t - \phi_r &= \phi_r' - \phi_t' \pm \pi\end{aligned}}$$

Analysis of the MZ interferometer

Use the notation for the fields as illustrated. Then



$$E_3 = r' E_1' + t\, E_2'$$

not B.S in, mode

$$E_4 = t' E_1' + r\, E_2'$$

Now

$$E_1' = E_1 e^{i\phi_A}.$$

where ϕ_A is the accumulated phase along path A, i.e.

$$\phi_A = \frac{2\pi L_A}{\lambda} + \phi_{MA}$$

A mirror

length of A

Similarly $E_2' = E_2 e^{i\phi_B}$

$$\Rightarrow \quad E_3 = r' E_1 e^{i\phi_A} + t\, E_2 e^{i\phi_B}$$

$$E_4 = t' E_1 e^{i\phi_A} + r\, E_2 e^{i\phi_B}.$$

Then $E_1 = r E_i$

$E_2 = t E_i$

Thus: $$E_3 = \{ r'r e^{i\phi_A} + t^2 e^{i\phi_B} \} E_i$$

$$E_4 = \{ t'r e^{i\phi_A} + rt e^{i\phi_B} \} E_i$$

and these can be use to determine intensities.

Exercise: For a 50-50 beam splitter, equal path lengths and identical mirrors, determine the intensity at the detectors (assume $\phi_t = \phi_{t'} = 0$)

$\phi_r = 0$

Answer: $r'r = \frac{1}{2} e^{i(\phi_{r'} + \phi_r)}$ $\quad t^2 = e^{2i\phi_t} \frac{1}{2}$ $\quad \phi_{r'} = \pi$

Now $\phi_A = \phi_B$

$$\Rightarrow E_3 = \frac{1}{2} e^{i\phi_A} \{ e^{i(\phi_{r'} + \phi_r)} + e^{2i\phi_t} \} E_i$$

But $\phi_{r'} = \pi \Rightarrow$ $E_3 = 0 \Rightarrow I_3 = 0$

$$E_4 = \frac{1}{2} e^{i\phi_A} \{ e^{i(\phi_r + \phi_{t'})} + e^{i(\phi_r + \phi_t)} \} E_i$$

$$= \frac{1}{2} e^{i\phi_A} \{ 1 + 1 \} E_i = e^{i\phi_A} E_i$$

$\Rightarrow I_4 = I_i$