Phys 473 2015

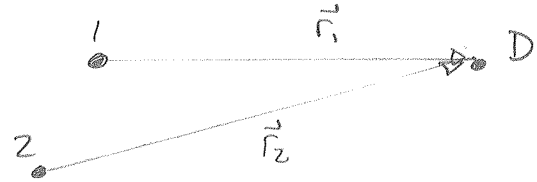
Lecture 23

HW is by Weds

Nobel prize

Interference between sources with the same frequency

For two sources of the same frequency the intensity at a detector is:



$$I = I_1 + I_2 + 2\,\vec{E}_{01}\cdot\vec{E}_{02}\cos\delta$$

where

$$\delta = \vec{k}_2\cdot\vec{r}_2 - \vec{k}_1\cdot\vec{r}_1 + \phi_2 - \phi_1$$

with the usual symbols for wave vector, phase, etc,... for each source apply.

Thin film interference

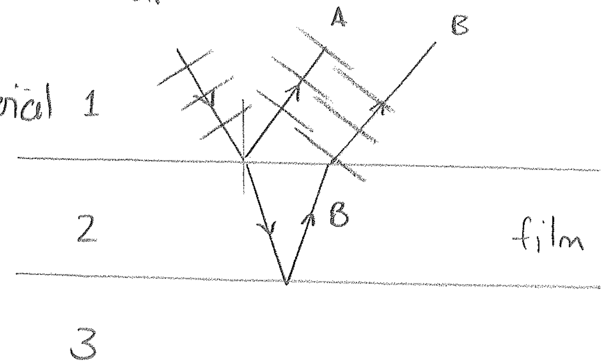
One application of this is to interference that results from multiple reflections off thin films. This ultimately explains optical phenomena such as:

1) irridesence (in minerals and animals)

2) rainbow patterns produced by oil films

Demo Uni Hannover website

The basic understanding of these is illustrated.



1) a wave in medium 1 is incident on the film
2) this is partly reflected (A) and partly transmitted (B)
3) wave B is partly transmitted into the incident medium.
4) the two reflected waves (A, B) interfere.

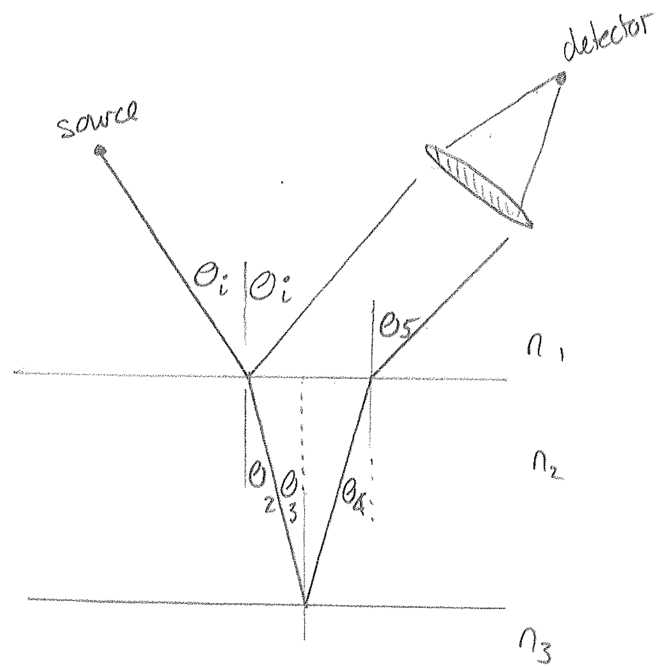
To describe how these interfere we need to describe:

1) the geometry of the rays
2) the phase difference between A and B.

The geometry is described via:

i) consider the source + detector and construct ray A. There is only one angle at which this can happen, θ_i as illustrated

ii) use Snell's law to determine the trajectory of the other ray B



<u>Exercise.</u> Use geometrical optics to relate, in a chain, $\theta_i, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ assume that the film has parallel sides.

<u>Answer:</u> $n_1 \sin\theta_i = n_2 \sin\theta_2$

$$\Rightarrow \quad \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_i$$

By geometry $\theta_3 = \theta_2$ and $\theta_4 = \theta_2$.

Then $n_2 \sin\theta_4 = n_1 \sin\theta_5$

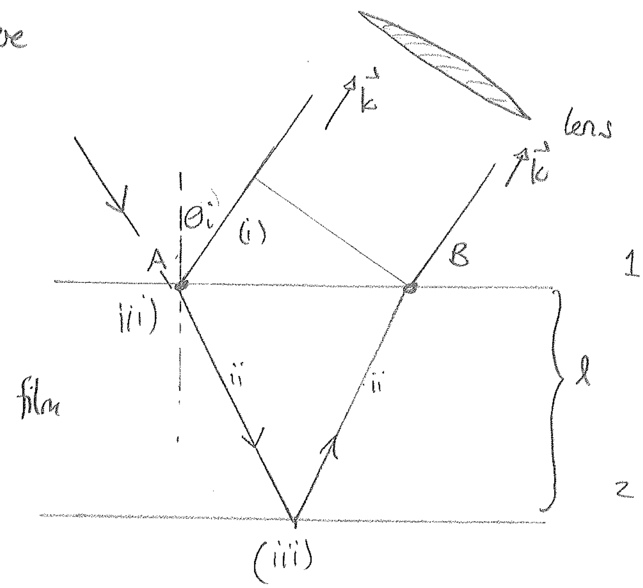
$$\Rightarrow \quad n_2 \sin\theta_2 = n_1 \sin\theta_5$$

$$\Rightarrow \quad n_1 \sin\theta_i = n_1 \sin\theta_5 \quad \Rightarrow \quad \theta_i = \theta_5$$

Thus for a parallel sided film $\theta_i = \theta_5$. and the <u>two reflected beams emerge parallel to each other</u>. Thus the wavevectors are the same. The analysis can be modified by reconsidering the situation in terms of two sources at the interface between the film and the incident medium.

$\rightarrow$ wave vector in 1

We can now reanalyze the situation by considering interference between two sources at A and B that produce plane waves with the same wavevector and frequency. We consider how these interfere at an infinitely distant detector. In practice this can be attain at a finite distance with an intermediate converging lens. The crucial factor in this will be the phase difference between the wave from A and that from B. There are three factors:

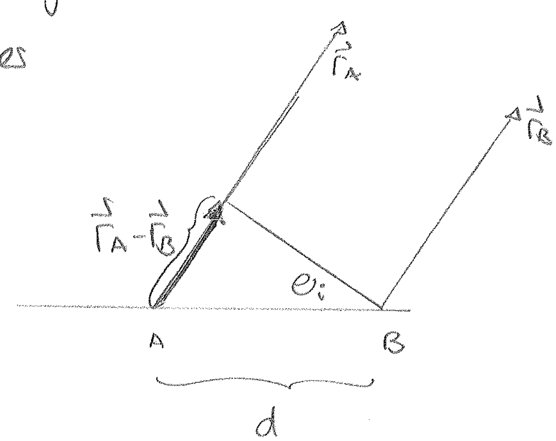


i) different distances traveled in medium 1

ii) the lag at the source B relative to A as a result of the extra distance traveled by the wave passing through B.

iii) phase shifts from reflections at the interfaces

These result in



$$\delta = \underbrace{\vec{k}\cdot(\vec{r}_A - \vec{r}_B)}_{\substack{(i)\\ \text{contributes here}}} + \underbrace{\phi_A - \phi_B}_{\substack{(ii)\ (iii)\\ \text{contribute here.}}}$$

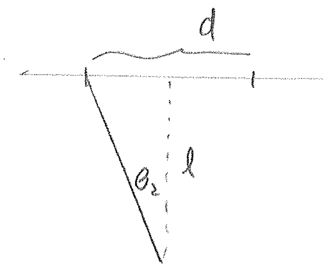
<u>Contribution (i)</u> Let d be the distance between A, B. Then

$$\vec{k}\cdot(\vec{r}_A - \vec{r}_B) = k(r_A - r_B)$$

and $\dfrac{r_A - r_B}{d} = \sin\theta_i \Rightarrow \vec{k}\cdot(\vec{r}_A - \vec{r}_B) = kd\sin\theta_i$

However, d depends on the film thickness l. Geometry yields.

$$\frac{d/2}{l} = \tan\theta_2 = \quad d = 2\,l\tan\theta_2$$



Thus: $\quad \vec{k}\cdot(\vec{r}_A - \vec{r}_B) = 2kl\sin\theta_i\tan\theta_2$

We now introduce a subtle point. We will work with the wavelength of light in a vacuum, let λ_0 be the wavelength in a vacuum. Then we know that in any medium, with index n, the wavelength is $\lambda = \lambda_0/n$.

Exercise: Use this to rewrite $\vec{k}\cdot(\vec{r}_A - \vec{r}_B)$ in terms of λ_0.

Answer: $\quad k = 2\pi/\lambda \quad$ and $\quad \lambda = \lambda_0/n_1 \quad \Rightarrow \quad k = \frac{2\pi}{\lambda_0} n_1$

$$\Rightarrow \quad \vec{k}\cdot(\vec{r}_A - \vec{r}_B) = \frac{4\pi l}{\lambda_0} n_1 \sin\theta_i \tan\theta_2$$

This is the contribution (i).

Contribution (ii) The incident wave transmitted into the film suffers no phase shift on entering. However there is a time delay and this introduces a phase shift between A and B. To separate this from the reflection phase shift we write:

$$\phi_A - \phi_B = \underbrace{\Delta\phi_{path}}_{\text{contribution (ii)}} + \underbrace{\Delta\phi_{ref}}_{\text{(iii)}}.$$

We show that in general

$$\Delta\phi_{path} = -2\pi\, D/\lambda$$

where D is the path length and λ the wavelength difference.

Proof: Suppose the time to travel from A to B is Δt. Then the two emerging waves combine as (ignoring reflections)

$$\vec{E}_{0A}\cos(\vec{k}\cdot\vec{r}_A-\omega t)+E_{0B}\underbrace{\cos(\vec{k}\cdot\vec{r}_B-\omega(t+\Delta t))}_{\cos(\vec{k}\cdot\vec{r}_B-\omega t\ -\omega\Delta t)}$$

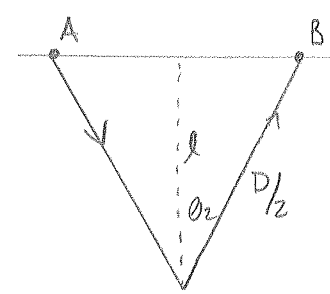
$$\Rightarrow \quad \Delta\phi_{path} = -\omega\Delta t \quad \text{and} \quad \omega = vk$$

$$\Rightarrow \quad \Delta\phi_{path} = -vk\Delta t = -kD \qquad \text{since } v\Delta t = D$$

$$\Rightarrow \quad \Delta\phi_{path} = -2\pi D/\lambda$$

$\blacksquare$

Exercise: Use this result and geometry to express $\Delta\phi$ in terms of θ_2, l, λ_0



Answer: $l/(D/2) = \cos\theta_2$

$$\Rightarrow \quad l = \frac{D}{2}\cos\theta_2$$

$$\Rightarrow \quad \frac{2l}{\cos\theta_2} = D$$

$$\Rightarrow \quad \Delta\phi_{path} = \frac{-2\pi}{\lambda}\frac{2l}{\cos\theta_2} = \frac{-4\pi l}{\lambda\cos\theta_2}$$

Now $\lambda = \lambda_0/n_2 \Rightarrow$ $\Delta\phi_{path} = \dfrac{-4\pi l}{\lambda_0\cos\theta_2}n_2$

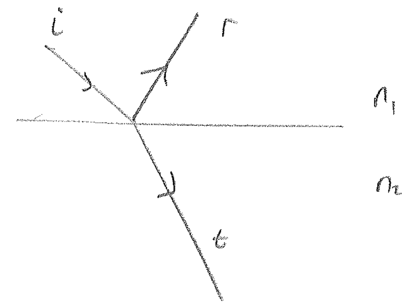
This gives contribution (ii).

Contribution (iii) If we return to boundary conditions for electric fields we can show that:

a) There is no phase shift for a transmitted wave

b) There is a π phase shift for a reflected wave if $n_2 > n_1$

c) There is no phase shift for a reflected wave if $n_2 < n_1$



We have to use these to get:

$$\boxed{\Delta\phi_{ref} = \text{reflection phase shift for A} - \text{reflection phase shift for B}}$$

We have to insert this by hand.

We can now combine all three contributions. We show that

$$\boxed{\delta = -\frac{4\pi l}{\lambda_0}\sqrt{n_2^2 - n_1^2 \sin^2\theta_i} + \Delta\phi_{ref}}$$

Proof: $\delta = \vec{k}\cdot(\vec{r}_A - \vec{r}_B) + \Delta\phi_{path} + \Delta\phi_{ref}$

$$= \frac{4\pi l}{\lambda_0} n_1 \sin\theta_i \tan\theta_2 - \frac{4\pi l}{\lambda_0}\frac{n_2}{\cos\theta_2} + \Delta\phi_{ref}$$

$$= \frac{4\pi l}{\lambda_0}\left\{ n_1 \sin\theta_i \frac{\sin\theta_2}{\cos\theta_2} - \frac{n_2}{\cos\theta_2}\right\} + \Delta\phi_{ref}$$

But, by Snell's law $n_2 \sin\theta_2 = n_1 \sin\theta_i$ and then

$$\delta = \frac{4\pi l}{\lambda_0}\frac{1}{\cos\theta_2}\left\{ n_2 \sin^2\theta_2 - n_2 \right\} + \Delta\phi_{ref}.$$

$$= \frac{4\pi l}{\lambda_0}\frac{n_2}{\cos\theta_2}\left(\sin^2\theta_2 - 1\right) + \Delta\phi_{ref}$$

$$\Rightarrow \quad \delta = + \frac{4\pi l}{\lambda_0} n_2 \frac{(-\cos^2\theta_2)}{\cos\theta_2} + \Delta\phi_{ref}$$

$$= - \frac{4\pi l}{\lambda_0} n_2 \cos\theta_2 + \Delta\phi_{ref}$$

$$= - \frac{4\pi l}{\lambda_0} n_2 \sqrt{1 - \sin^2\theta_2} + \Delta\phi_{ref}$$

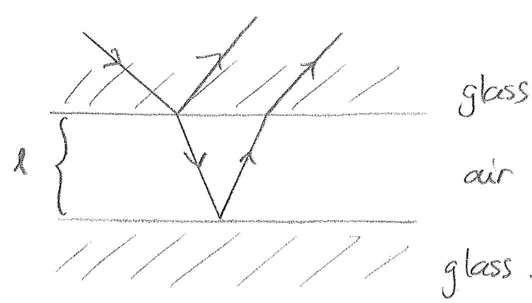
$$= - \frac{4\pi l}{\lambda_0} \sqrt{n_2^2 - \underbrace{n_2^2 \sin^2\theta_2}_{= n_1^2 \sin^2\theta_i}} + \ldots$$

$$= - \frac{4\pi l}{\lambda_0} \sqrt{n_2^2 - n_1^2 \sin^2\theta_i} + \Delta\phi_{ref} \qquad \boxtimes$$

Application: Interference fringes between glass plates.

The pair of reflected rays is as illustrated. Then $n_1 > n_2$

$n_2 < n_3$



gives $\quad \Delta\phi_{ref} = 0 - \pi$

$$\Rightarrow \quad \delta = -\pi \left\{ \frac{4l}{\lambda_0} \sqrt{n_2^2 - n_1^2 \sin^2\theta_i} + 1 \right\}$$

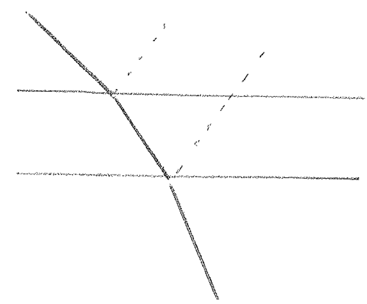
For normal incidence $\theta_i = 0 \Rightarrow \quad \delta = -\pi \left\{ \frac{4 l n_{air}}{\lambda_0} + 1 \right\}$

$\Rightarrow$ various thicknesses give $\delta = \pm\pi, \pm 3\pi$ which give dark fringes

as $l \rightarrow 0 \quad \delta \rightarrow -\pi \Rightarrow$ dark fringe.

Antireflection coating

When there is destructive interference between the reflected waves then the light will be transmitted. This can be arranged with $n_1 < n_2 < n_3$. Then



$\Delta\phi_{ref} = 0$

$$\Rightarrow \quad \delta = -\frac{4\pi l}{\lambda_0}\sqrt{n_2^2 - n_1^2 \sin^2\theta_i}$$

and when $\delta = (2m+1)\pi$ we get destructive interference and all light is reflected This is when:

$$(2m+1)\cancel{\pi} = -\frac{4\cancel{\pi} l}{\lambda_0}\sqrt{\quad} \quad \Rightarrow \quad -\frac{(2m+1)}{4} = \frac{l}{\lambda_0}\sqrt{n_2^2 - n_1^2\sin^2\theta_i}$$

gives the the necessary condition.

Non-total internal reflection requirement.

What if $n_1 \sin\theta_i > n_2$? Then the main result appears invalid. Snell's law states that $n_1\sin\theta_i = n_2\sin\theta_2 \Rightarrow \frac{n_1\sin\theta_i}{n_2} = \sin\theta_2$ and so the requirement is $\sin\theta_2 > 1$. This will occur when $n_1 > n_2$ and the incident ray is beyond the critical angle - then there will be no source B. The critical angle occurs when $\theta_2 = 90° \Rightarrow n_1\sin\theta_c = n_2$. If $\theta_1 > \theta_c$ then $n_1\sin\theta_1 > n_2$. So

Interference only occurs when $n_1\sin\theta_i < n_2$
If this is not satisfied, total internal reflection occurs at $1 \to 2$ and no light passes into 2.