Phys 473 2015

Lecture 20

HW due by 5pm

Lens Combinations

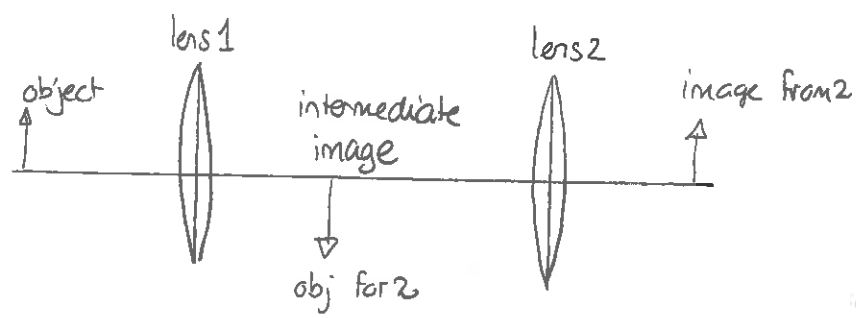
Lenses are often used in combination with other lenses. Reasons for this include:

a) increasing magnification

b) reduction of aberrations

The basic method of analysis is.

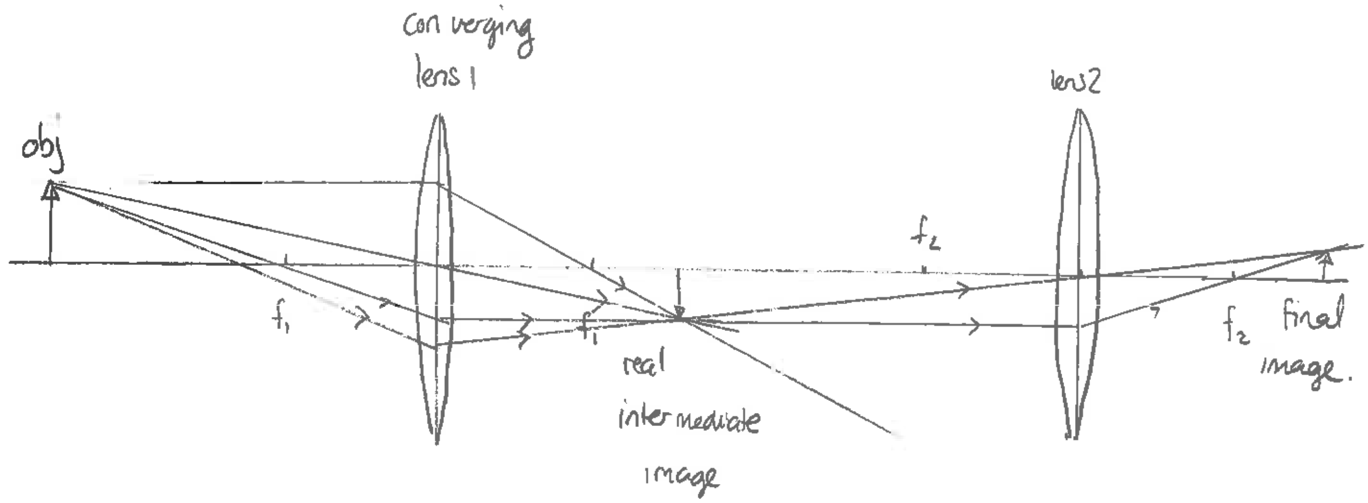
1) lens 1 produces an image of the object (an intermediate image)



2) lens 2 uses the intermediate image as an object and produces a final image.

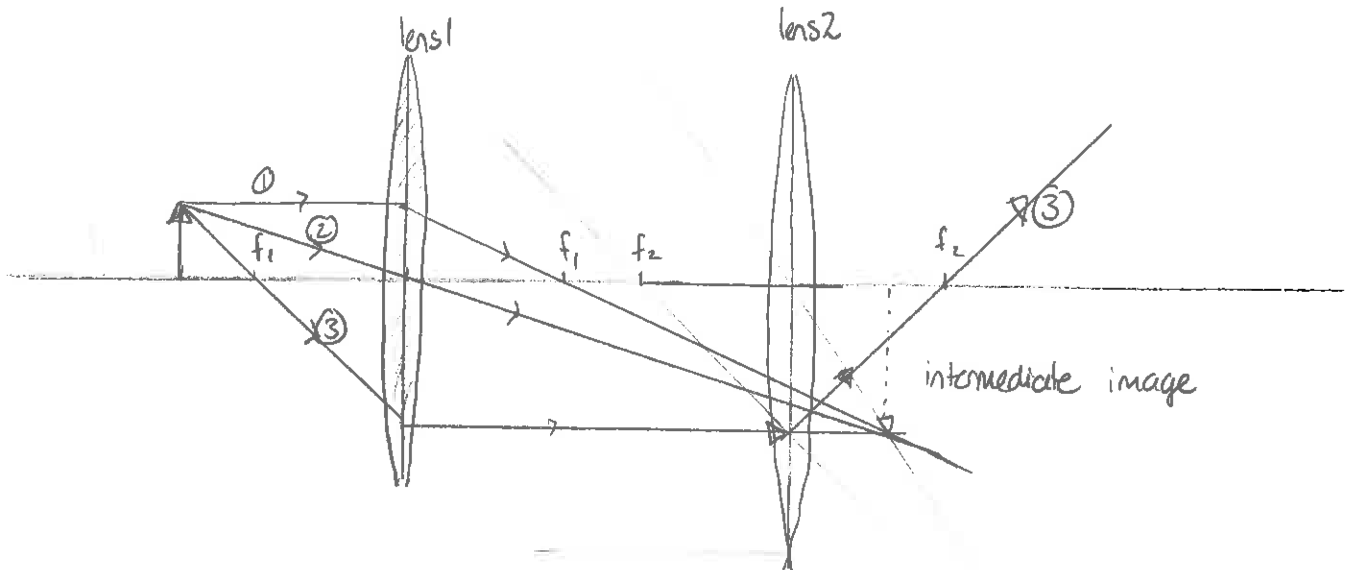
We can use the lens equation for each individual lens to assess the entire process. There are two cases to consider.

1) the intermediate image is formed to the left of lens 2. Whether the image is real or virtual, it serves as a real object for the second lens.



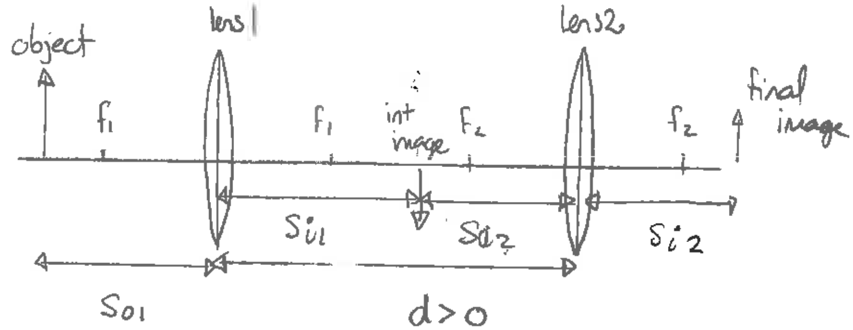
Here the light rays actually pass through the real intermediate image. Thus it acts as exactly any real object would for the second lens. Even if the intermediate image were real then the rays would appear to come from the intermediate image and then pass through the second lens. There is no difference with a real source object. In this case we require that the distance S_{o2} from the intermediate image to lens 2 is positive

2) the intermediate image is right of lens 2. This can only occur if lens 1 is a converging lens.



Here rays ①, ② ③ do not actually reach the intermediate image before they are refracted by the second lens. For example the actual path of ray ③ is illustrated. So the intermediate image acts as a virtual object for the second lens We need $S_{o2} < 0$ here

These can all be accounted for by considering the following set up.



1) the signs of the focal lengths are determined by lens properties

2) $S_{o1} > 0$ by convention

3) $d > 0$ by convention

4) $S_{o2} = d - S_{i1}$. If 1 produces a virtual image then clearly $S_{o2} > 0$ and we can proceed without any difficulty.

If 1 produces a real image then $S_{i1} > 0$ and if it is between the lenses then $d - S_{i1} > 0$ and this is again the usual real object.

If $S_{i1} > d$ then the intermediate image is right of lens 2. Then the image production for 2 is reversed. and $S_{o2} < 0$.

Then

$$\boxed{S_{i2} = \frac{d(S_{o1} - f_1) f_2 - f_1 f_2 S_{o1}}{d(S_{o1} - f_1) - (f_1 + f_2) S_{o1} + f_2 f_1}}$$

<u>Proof</u> For either lens $\frac{1}{S_i} + \frac{1}{S_o} = \frac{1}{f} \Rightarrow \frac{1}{S_i} = \frac{1}{f} - \frac{1}{S_o} = \frac{S_o - f}{f S_o}$

$$\Rightarrow S_i = \frac{f S_o}{S_o - f}$$

Thus

$$s_{i1} = \frac{f_1 s_{o1}}{s_{o1} - f_1}$$

and

$$s_{i2} = \frac{f_2 s_{o2}}{s_{o2} - f_2} = \frac{f_2 (d - s_{i1})}{(d - s_{i1}) - f_2}$$

$$= \frac{f_2 \left(d - f_1 s_{o1} / s_{o1} - f_1\right)}{d - f_1 s_{o1} / s_{o1} - f_1 - f_2}$$

$$= \frac{f_2 \left[d(s_{o1} - f_1) - f_1 s_{o1} \right]}{(d - f_2)(s_{o1} - f_1) - f_1 (s_{o1})} = \frac{d(s_{o1} - f_1) f_2 - f_1 f_2 s_{o1}}{d(s_{o1} - f_1) - (f_1 + f_2) s_{o1} + f_2 f_1}$$

Exercise: Suppose that the lenses are converging and arranged so that the focal points between them coincide. Determine an expression for s_{i2}.

Answer $d = f_1 + f_2 \quad \Rightarrow \quad s_{i2} = \frac{(f_1 + f_2)(s_{o1} - f_1) f_2 - f_1 f_2 s_{o1}}{(f_1 + f_2)(\cancel{s_{o1}} - f_1) - \cancel{(f_1 + f_2)} s_{o1} + f_1 f_2}$

$$= \frac{f_2^2 (s_{o1} - f_1) - f_1^2 f_2}{f_1^2 - \cancel{f_2 f_1 + f_1 f_2}}$$

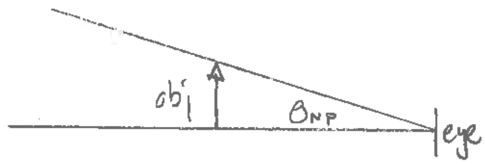
$$\Rightarrow \quad s_{i2} = \frac{f_2^2 s_{o1} - (f_1 + f_2) f_1 f_2}{f_1^2}$$

$$\Rightarrow \quad s_{i2} = \frac{f_2^2}{f_1^2} s_{o1} - \frac{f_2}{f_1} (f_1 + f_2)$$

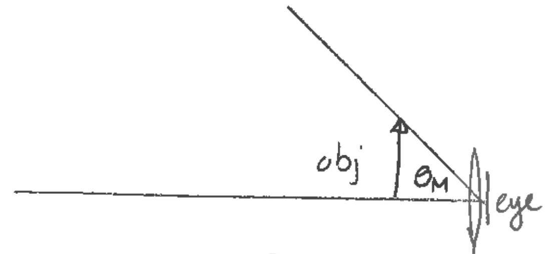
Clearly $s_{i2} > 0$ when $\frac{f_2}{f_1} s_{o1} > (f_1 + f_2) \iff s_{o1} > \frac{f_1}{f_2} (f_1 + f_2)$

Magnifier

The simplest optical instrument is a magnifier. This consists of a single converging lens which produces a virtual image. The eye views this at an apparently enlarged angle



no magnifier

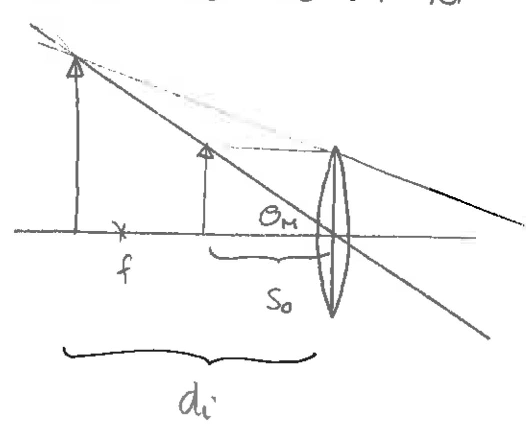


with magnifier.

The magnifier is held adjacent to the eye and we will assume zero separation between these. The aim is to be able to bring the object closer so as to increase the angle as illustrated. The magnifier's power hinges on the fact that there is a near point distance d_{NP} beneath which the eye cannot form clear images. Thus the maximum angular displacement of an object is when it is at the nearpoint. For an object of height h

$$\frac{h}{d_{NP}} = \tan \theta_{NP}$$

Inserting a lens allows one to bring the object closer and having the eye view the more distant virtual image. For this we need $S_o < f$ for the lens. Then using the diagram



$$\frac{h}{S_o} = \tan \theta_M.$$

The angular magnification is:

$$M_{ang} = \frac{\theta_M}{\theta_{NP}}$$

With $\tan\theta \approx \theta$ we get

$$M_{ang} = \frac{d_{NP}}{s_o}$$

What s_o is best? Clearly smaller is better but as s_o decreases the image approaches the lens. It cannot approach any closer than the near point. At this stage

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \Rightarrow \frac{1}{f} = \frac{1}{s_o} + \frac{1}{(-d_{NP})}$$

virtual image

$$\Rightarrow \frac{1}{s_o} = \frac{1}{f} + \frac{1}{d_{NP}}$$

So

$$\boxed{M_{ang} = 1 + \frac{d_{NP}}{f}}$$

gives the maximum magnification.

<u>Demo</u>. Do this with magnifier

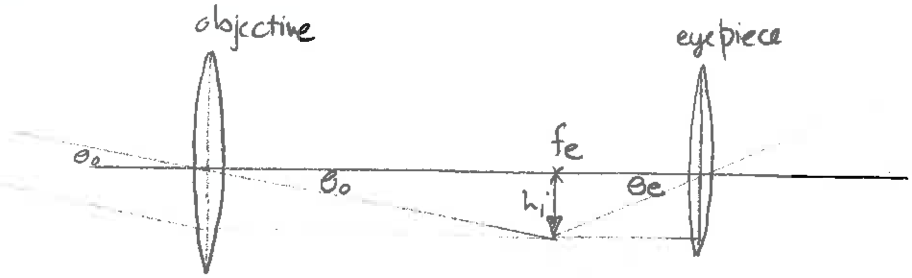
- measure NP
- compare against grid. → measure ~~d~~ ~~s_i~~? $s_o = ?$

Relaxed viewing magnification occurs when the image is infinitely distant. Then $s_i \to \infty$ and $\frac{1}{s_o} \to \frac{1}{f}$ gives:

> Relaxed eye magnification occurs when $s_o \approx f$ (slightly less) and then
>
> $$M_{ang} = \frac{d_{NP}}{f}$$

Telescope:

A telescope consists of two converging lenses. That closest to the eye (called the eyepiece) functions as a magnifier. Thus the intermediate image is just within the focal point of the eyepiece. The objective must form an image at this location. The magnification is again



$$M_{ang} = \frac{\theta_e}{\theta_o}$$

For viewing distant objects $s_o \to \infty$ and $\frac{1}{s_i} = \frac{1}{f_o} \Rightarrow s_i = f_o$.

So we need the focal points of the lens + eyepiece to coincide. Then using the diagram

$$\frac{h_i}{f_o} \approx \theta_o \qquad \frac{h_i}{f_e} \approx \theta_e \qquad \Rightarrow \qquad \frac{\theta_e}{\theta_o} = \frac{f_o}{f_e}$$

So $$M_{ang} = \frac{f_o}{f_e}$$

Demo: Do with two PASCO lenses.