Phys 473 2015

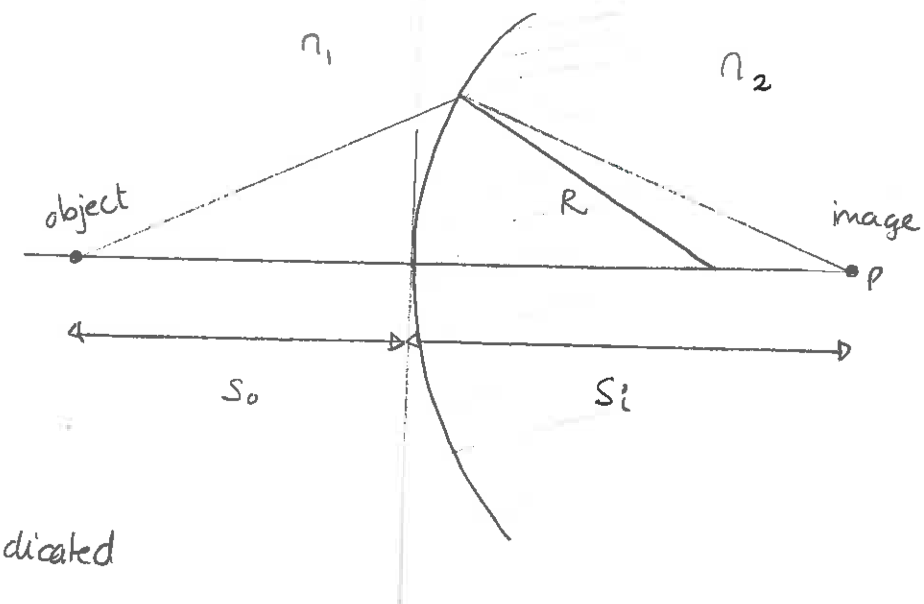
Lecture 19

Thurs: Seminar

Spherical Refracting Surfaces

For a spherical refracting surface, we saw that if angles were relatively small then

$$\boxed{\frac{n}{S_i} + \frac{1}{S_o} = \frac{n-1}{R}}$$



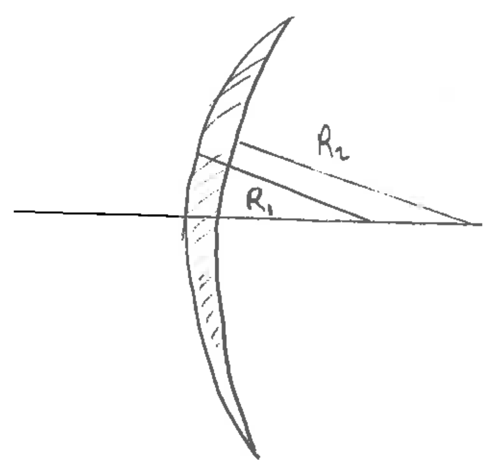
where S_i, S_o are the indicated distances and

$$n = n_2/n_1$$

the ratio of the indices of refraction.

Then for a lens with two surfaces as illustrated,

$$\boxed{\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}}$$



where

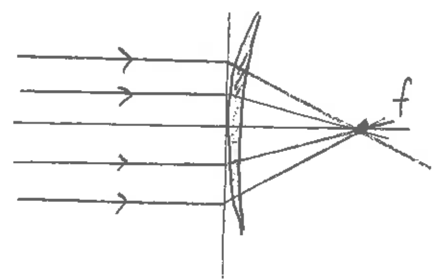
$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

and $n = n_{lens}/n_{medium}$

We can again interpret the focal length by considering an infinitely distance source. Then $s_o \to \infty \Rightarrow s_i \to f$. We then define:

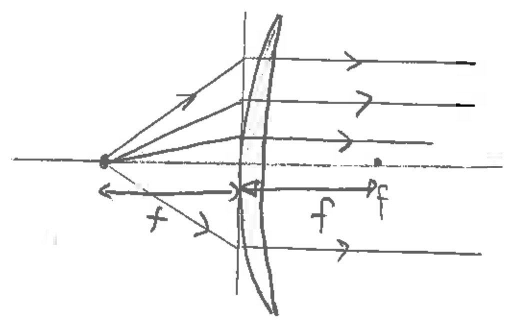
> The focal point of a lens is the point through which all rays from an infinitely distance source pass. The focal length is the distance from the lens to the focal point

If the infinitely distant object is on the optical axis, then the rays travel parallel to the axis as illustrated.



We can describe a focal point to the left of the lens by considering $s_i \to \infty$. Then $s_o = f$. Thus rays that emerge from this point to the left all pass parallel on the right. Such left and right focal points are equally distant from the lens. So

> Regardless of lens curvature the focal point on the right is equally distant from the focal point on the left.



Notice that the formula for the focal length assumes that the center of curvature for each surface is right of the lens. In this case the radii are positive. In general the formula works if:

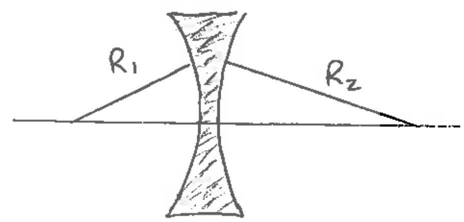
> $R > 0$ when the center of curvature is right of the lens
>
> $R < 0$ " " " " " left of the lens
>
> (object is always left in this set up).

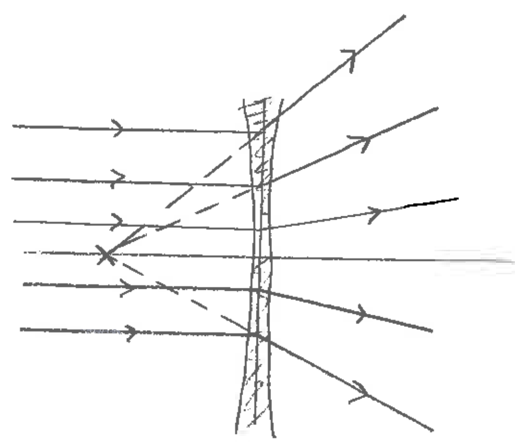
In the illustrate lens $0 < R_1 < R_2 \Rightarrow \frac{1}{R_1} - \frac{1}{R_2} > 0 \Rightarrow f > 0$.
This fact, that $f > 0$, is typical for converging lenses.

Demo: Use PASCO light source
- determine f
- using $n_{lens} = 1.49$
  $n_{air} = 1.00$

and radius of curvature, calculate f.

If $R_1 < 0$ and $R_2 > 0$ then the lens will be bi-concave and have a negative focal length. This lens is diverging. One way of understanding this is to consider rays that approach parallel to the optical axis. Then $s_o = \infty$ and $s_i < 0$ will indicate an apparent image formed to the left.





We will assess image production by using the thin lens equation, and also ray tracing. Ray tracing for lenses involves

1) a ray parallel to the optical axis and passing through a focal point (converging) or appearing to emanate from a focal point (diverging)

2) a ray passing through the near focal point and emerging parallel.

3) a ray passing directly through the center.

Image formation with lenses

1) Convex lenses. Suppose $R_2 > R_1 > 0$. Then $f > 0$, and the lens is converging.

Exercise a) Use the thin lens equation to determine conditions that give an image formed right of the lens. Repeat for left of the lens.

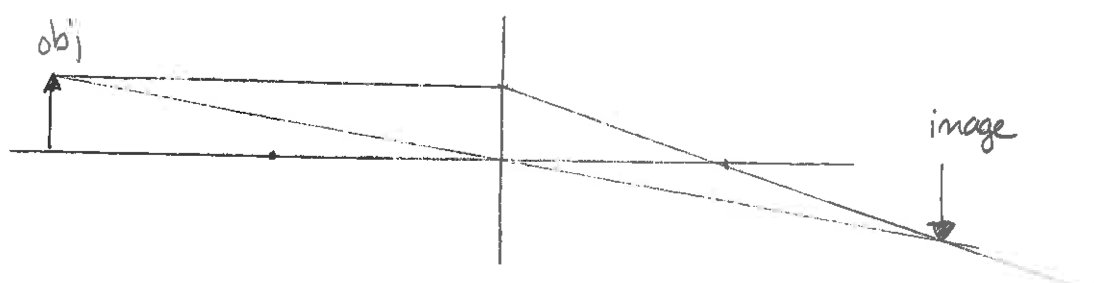
b) Trace rays for both cases and use the results to describe whether the image is upright, inverted and real/virtual

Answer: a) $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \Rightarrow \frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}$

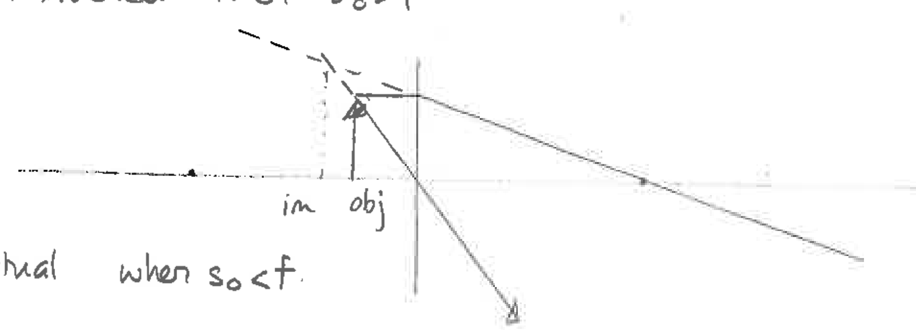
left $\Rightarrow s_i > 0 \Rightarrow \frac{1}{f} > \frac{1}{s_o} \Rightarrow f < s_o$ (object is beyond focal pt)

right $\Rightarrow s_i < 0 \Rightarrow \frac{1}{f} < \frac{1}{s_o} \Rightarrow f > s_o$ (object is between lens + focal pt)

b).



real + inverted when $s_o > f$



upright + virtual when $s_o < f$.

Exercise: a) Suppose that $s_o > f$. As s_o increases does the image size increase or decrease? Does the image move closer or further away?

b) Suppose that $s_o < f$. Repeat previous question.

Answer: a) $\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}$ $\quad s_o \uparrow \Rightarrow \frac{1}{s_o} \downarrow \Rightarrow -\frac{1}{s_o} \uparrow$

$$\Rightarrow \frac{1}{s_i} \uparrow \Rightarrow s_i \downarrow$$

If s_o increases s_i decreases $\Rightarrow$ closer

By the ray diagram image size decreases.

b) as s_o decreases the image clearly moves closer and its height decreases.

The magnification of the image is

$$\boxed{M = \frac{h_i}{h_o}}$$

where h_i = height of image

h_o = " " object

Geometrical reasoning gives

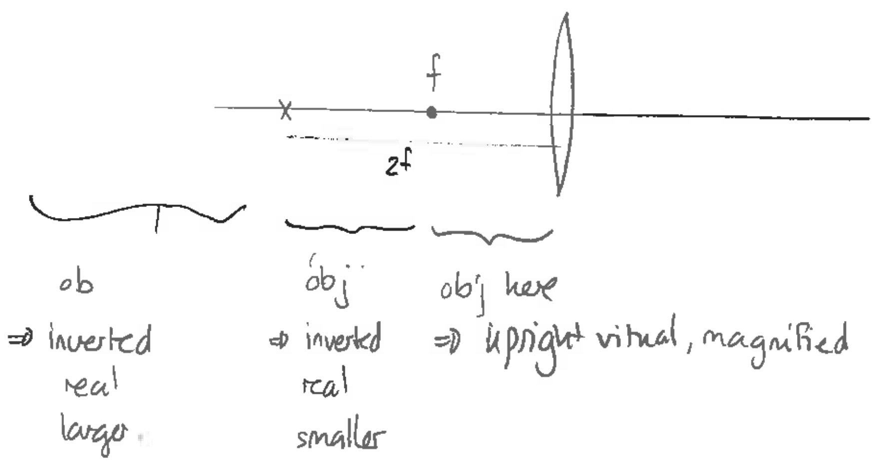
$$\boxed{M = -s_i/s_o}$$

Exercise: Determine when a convex lens gives a magnification of -1

Answer: Need $+S_i = S_o$

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f} \Rightarrow \frac{2}{S_o} = \frac{1}{f} \Rightarrow S_o = 2f$$

So we get:



2) Concave lenses

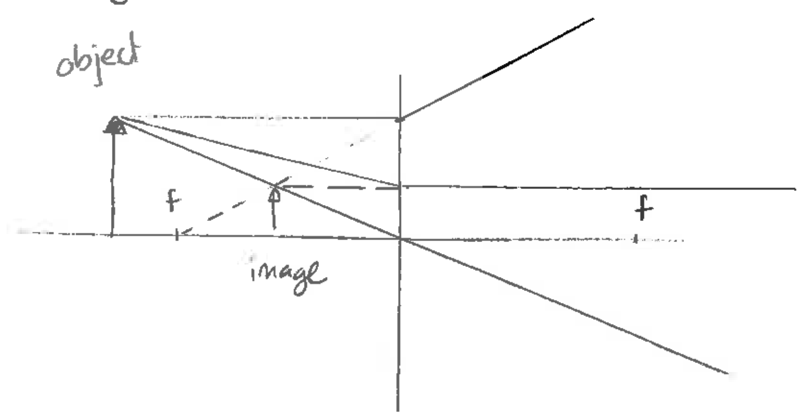
Suppose that $R_1 < 0$. Then (unless $R_2 < 0$ and $R_2 < R_1$) the lens will have a negative focal length.

Exercise: a) Show that regardless of s_o, an image will always be formed to the left of the lens.

b) Show that the image will always be upright and smaller than the object. Describe what happens to the image size and location as the object approaches the lens.

Answer a) $\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}$ is clearly negative

b) Trace rays.



Clearly the image is upright. Adjusting the object adjust the diagonal ray.

obj closer = steeper ⇒ image closer + larger

So we have

lens	s_o	image
concave	any	upright, virtual, smaller, left
convex	$s_o < f$	upright, virtual, larger, left
	$f < s_o < 2f$	inverted, real, smaller, right
	$2f < s_o$	" " larger, "