Phys 473 2015

Lecture 13

HW 9 available

Reflection + Transmission: Perpendicular Polarization

For perpendicular polarization, electric fields at an interface are related as:

$$E_{or}^{\perp} = r^{\perp} E_{oi}^{\perp} \qquad r^{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$E_{ot}^{\perp} = t^{\perp} E_{oi}^{\perp} \qquad t^{\perp} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

perpendicular polarization

and these must be combined with Snell's Law $n_i \sin\theta_i = n_t \sin\theta_t$. The resulting expressions for the Fresnel amplitude ratios are:

$$r^{\perp} = \frac{\sqrt{1-\sin^2\theta_i} - \sqrt{\left(\frac{n_t}{n_i}\right)^2 - \sin^2\theta_i}}{\sqrt{1-\sin^2\theta_i} + \sqrt{\left(\frac{n_t}{n_i}\right)^2 - \sin^2\theta_i}}$$

$$t^{\perp} = 2\,\frac{\sqrt{1-\sin^2\theta_i}}{\sqrt{1-\sin^2\theta_i} + \sqrt{\left(\frac{n_t}{n_i}\right)^2 - \sin^2\theta_i}}$$

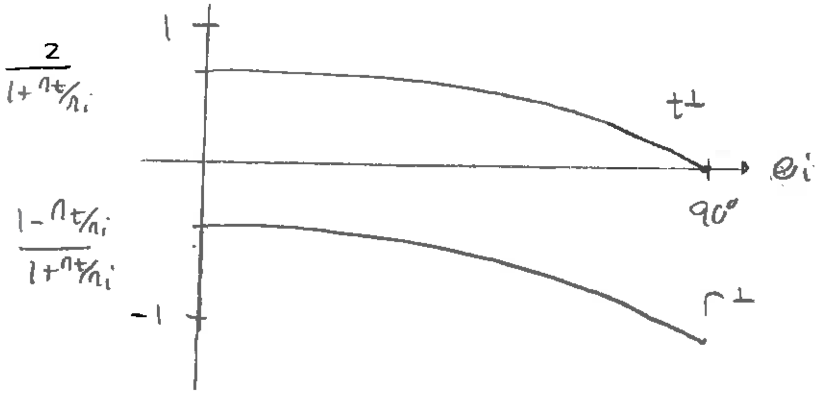
The way that these behave depends on n_i vs n_t.

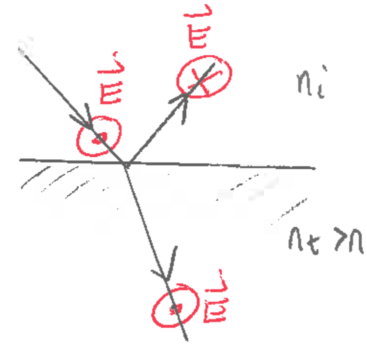
Incident in a faster medium ($n_i < n_t$)

Analysis reveals that when $n_i < n_t$ these equations are valid for all θ_i. Also it shows

$r^{\perp}$ is negative and decreases from $\frac{n_i - n_t}{n_i + n_t}$ to -1 as θ_i increases
$t^{\perp}$ is positive and decreases from $\frac{2n_i}{n_i+n_t}$ to 0 as θ_i increases

For any given n_i and n_t these can be plotted vs θ_i





We see that

$\boxed{\text{Unless } n_i = n_t \text{ or } \theta_i = 90^\circ \text{ there is always some reflected light and some transmitted light}}$

Incident in a slower medium ($n_t < n_i$)

The equations are only valid up to the critical angle $\theta_c = \sin^{-1}\left(\frac{n_i}{n_t}\right)$

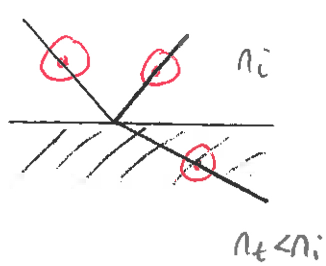
So for $\theta_i < \theta_c$ we can show:

$r^\perp$ is positive and increases with θ_i from $\frac{n_i - n_t}{n_i + n_t}$ to 1 (at θ_c)	
$t^\perp$ " positive and increases. with θ_i from $\frac{2n_i}{n_i + n_t}$ to 2 (at θ_c)	

We see that

$\boxed{\text{Unless } n_i = n_t \text{ then there is always some reflected and some transmitted light. if } \theta_i < \theta_c.}$

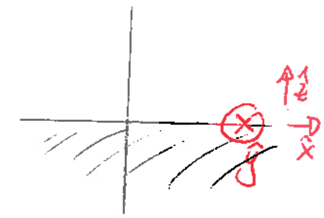
only up to θ_c!



Phase changes

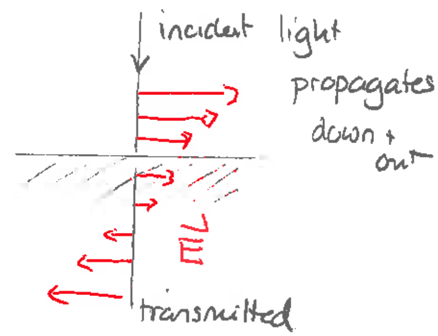
In both cases we see that the transmission ratio is positive. Recall that the analysis used

$$\vec{E} = \vec{E}_o e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$



and for perpendicular polarization $\vec{E}_o = E_{oy}\hat{y}$. The sign of E_o then describes the direction in which the $\vec{E}$ field points. It follows that $t > 0$ implies that the transmitted $\vec{E}$ field points in the same direction as the incident $\vec{E}$ field. This is illustrated in the previous diagrams. We see that the fields are in phase.

However depending on the relative indices $r^{\perp}$ is negative or positive and E_{or} could be reversed relative to E_{oi}. Then

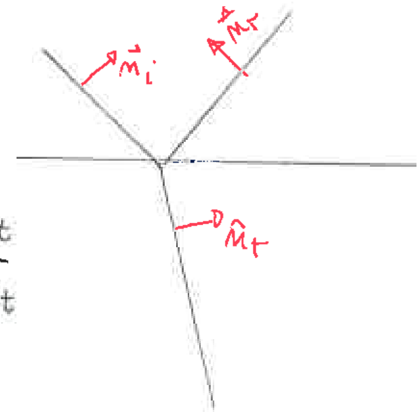


If $n_t > n_i$ the reflected electric field has an orientation opposite to that of the incident field. There is a 180° phase shift

If $n_t < n_i$ the reflected electric field points in the same direction as the incident field. There is no phase shift whenever $\theta_i < \theta_c$

Reflection + Transmission: Parallel Polarization

In this case the Fresnel equations refer to components along the indicate unit vector directions. Then



$$E_{or}^{\parallel} = r^{\parallel} E_{oi}^{\parallel} \qquad r^{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$E_{ot}^{\parallel} = t^{\parallel} E_{oi}^{\parallel} \qquad t^{\parallel} = \frac{2 n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

We saw that at the Brewster angle

$$\theta_B = \tan^{-1} \frac{n_t}{n_i}$$

there is no reflection; We can rewrite the above and carry out a similar analysis. We find the same critical angle and

If	$n_i > n_t$	then	$r^{\parallel}$	is	an	increasing	function of $\theta_i < \theta_c$
If	$n_i < n_t$	then	$r^{\parallel}$	〃	a	decreasing	function of θ_i
If	$n_i > n_t$	then	$t^{\parallel}$	〃	an	increasing	function of $\theta_i < \theta_c$
If	$n_i < n_t$	then	$t^{\parallel}$	〃	a	decreasing	〃 〃 θ_i

Exercise: a) Suppose $n_i > n_t$. Find extreme values of r'' and t''. Are they positive or negative?

b) " $n_i < n_t$ " " " " r'' and t''. Are they positive or negative?

Answer: a) Since r'' is an increasing function of θ_i the minimum is attained when $\theta_i = 0$ and the max when $\theta_i = \theta_c$.

Now $\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i$. So $\theta_i = 0° \Rightarrow \theta_t = 0°$

So $r'' = \frac{n_t - n_i}{n_i + n_t} < 0$ is minimum.

Then the max is attained when $\theta_i = \theta_c$ and here $\theta_t = 90°$. So $r'' = 1$

Thus $\frac{n_t - n_i}{n_i + n_t} < r'' < 1$

The minimum of t'' is attained when $\theta_i = 0 \Rightarrow t'' = \frac{2n_i}{n_i + n_c}$

The maximum of t'' is attained when $\theta_i = \theta_c$. This gives $\theta_t = 90°$ and $t'' = 2$. So

$$\frac{2n_i}{n_i + n_c} < t'' < \frac{2n_i}{n_t}$$

b) r'': max when $\theta_i = 0 \Rightarrow r'' = \frac{n_t - n_i}{n_t + n_i} > 0$

min when $\theta_i = 90° \Rightarrow r'' = -1 < 0$

and $r'' < 0$

$$-1 < r'' < \frac{n_t - n_i}{n_t + n_i}$$

t'': max when $\theta_i = 0 \Rightarrow t'' = \frac{2n_i}{n_i + n_t}$

min when $\theta_i = 90° \Rightarrow t'' = 0$

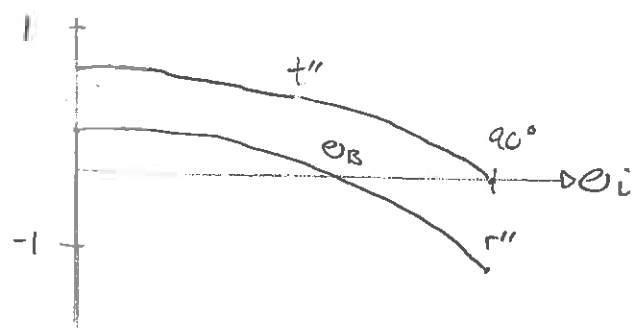
We can summarize as follows for <u>parallel polarization</u>

<u>Incident in a faster medium ($n_i<n_t$)</u>

The equations are valid for all θ_i.

> r'' decreases from $\frac{n_t-n_i}{n_i+n_t}>0$ to -1. At some point $r''=0$ (Brewster angle)
>
> t'' is always positive and decreases (as θ_i increases) from $\frac{2n_i}{n_i+n_t}<1$ to 0.

Plotting would yield

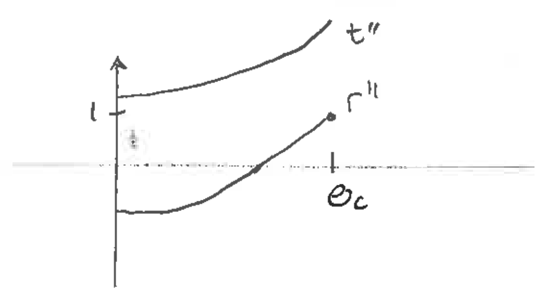


<u>Incident in a slower medium ($n_i>n_t$)</u>

The equations are valid for $\theta_i<\theta_c$

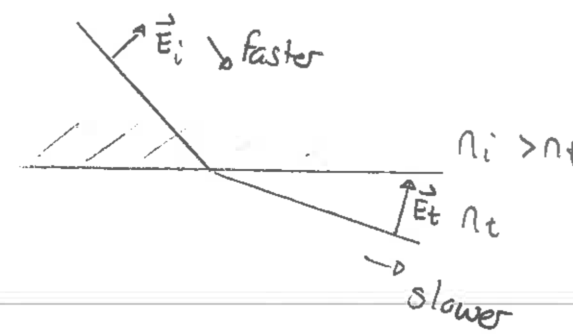
> r'' increases from $\frac{n_t-n_i}{n_i+n_t}<0$ to 1. At some point $r''=0$ (Brewster angle)
>
> t'' increases from $\frac{2n_i}{n_i+n_t}>\frac{2n_i}{n_t}$ to 2 at θ_c.

Plotting would give.



Intensity relationships for reflection and transmission

It appears that whenever the incident wave is in a slower medium, the transmitted wave will have an electric field whose magnitude is larger than that of the incident wave. It would appear that this results in enhanced intensity or irradiance. How could this be possible?



Recall that the irradiance, which is the energy transported by the wave per unit area per second is

$$I = \frac{1}{2}\epsilon v E_0^2$$

where ϵ is the permittivity of the medium and v the speed in the medium. The key is that both of these affect the intensity as well as the electric field. Note that

$$1/\sqrt{\epsilon\mu_0} = v \Rightarrow \frac{1}{\epsilon\mu_0} = v^2 \Rightarrow \quad \epsilon = \frac{1}{\mu_0 v^2}$$

But $\quad \epsilon_0 = \frac{1}{\mu_0 c^2} \Rightarrow \mu_0 = \frac{1}{\epsilon_0 c^2}$ and together these give:

$$\epsilon = \epsilon_0 \frac{c^2}{v^2} = \epsilon_0 n^2$$

Thus

$$I = \frac{1}{2} n^2 \epsilon_0 v E_0^2$$

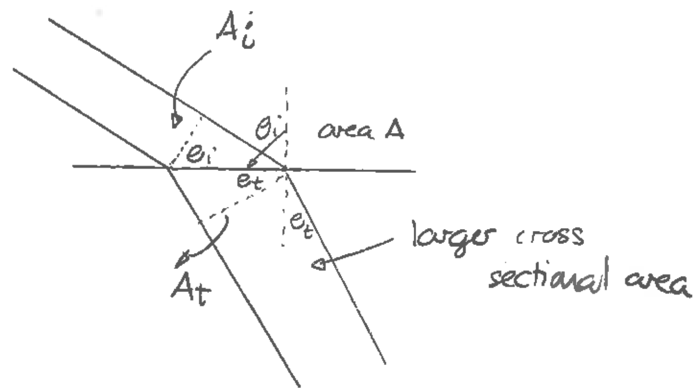
For larger index of refraction both $\frac{1}{n^2}$ and v are smaller. Thus the rate at which energy is transported is reduced. This will offset any enhancement in electric fields

Now $v = \frac{c}{n}$ gives:

$$I = \frac{1}{2} \epsilon_0 c\, n E_0^2$$

An additional complication is that upon transmission a beam is distributed over a different area.

Looking at the diagram we see that the cross section area of the incident beam A_i and that of the transmitted beam A_t are related using



$$\frac{A_i}{A} = \cos\theta_i \qquad \frac{A_t}{A} = \cos\theta_t$$

$$\Rightarrow \frac{A_t}{A_i} = \frac{\cos\theta_t}{\cos\theta_i}$$

We then define the transmittivity as

$$\boxed{T = \frac{\text{transmitted power}}{\text{incident power}} = \frac{I_t A_t}{I_i A_i}}$$

Then

$$T = \frac{I_t}{I_i} \frac{\cos\theta_t}{\cos\theta_i} = \frac{\cancel{\frac{1}{2}} \cancel{\epsilon_0} \cancel{c}\, n_t E_{0t}^2 \cos\theta_t}{\cancel{\frac{1}{2}} \cancel{\epsilon_0} \cancel{c}\, n_i E_{0i}^2 \cos\theta_i}$$

$$\Rightarrow T = t^2 \frac{n_t \cos\theta_t}{n_i \cos\theta_i}$$

Similarly we define the reflectivity as

$$\boxed{R = \frac{\text{reflected power}}{\text{incident power}} = \frac{I_r A_r}{I_i A_i}}$$

and a similar geometric argument gives:

$$R = \frac{I_r}{I_i} = r^2 \quad \Rightarrow \quad R = r^2$$

Exercise For perpendicular polarization, compute R, T.

Answer: $R = r^2 = \left(\frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}\right)^2$ and by the previous results

$\left(\frac{n_i - n_t}{n_i + n_t}\right)^2 \leq R \leq 1$ in both cases

$$T = t^2 = \frac{4 n_i^2 \cos^2\theta_i}{(n_i \cos\theta_i + n_t \cos\theta_t)^2} \frac{n_t \cos\theta_t}{n_i \cos\theta_i}$$

$$T = \frac{4 n_i n_t \cos\theta_i \cos\theta_t}{(n_i \cos\theta_i + n_t \cos\theta_t)^2}$$

We will later explore the properties of this