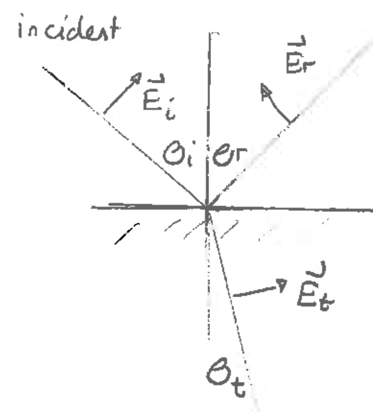
Phys 473

2015 Lecture 12

Next HW due Weds

Fresnel ratios



We saw that at an interface the directions of the reflected + transmitted waves are given by

$\theta_r = \theta_i$ (law of reflection)

$n_i \sin\theta_i = n_t \sin\theta_t$ (Snell's Law)

The electric fields are described generically as

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

Then boundary conditions at the interface yield

Perpendicular polarization "s polarization"

$$E_{or}^\perp = r^\perp E_{oi}^\perp \qquad r^\perp = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}$$

$$E_{ot}^\perp = t^\perp E_{oi}^t$$

$$t^\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$$

Parallel polarization "p polarization"

$$E_{or}^\parallel = r^\parallel E_{oi}^\parallel \qquad r^\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_i\cos\theta_t + n_t\cos\theta_i}$$

$$E_{ot}^\parallel = t^\parallel E_{ot}^\parallel$$

$$t^\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$$

We now explore this for various cases. Of interest are the extremes where ratios yield zero or one.

Perpendicular polarization

Consider the perpendicular polarization case.

Exercise 1: a) Determine a condition for no transmission

b) Use Fresnel's equations to determine a condition which gives $r^{\perp}=0$

c) Combine Snell's law with the previous result to determine whether the condition can be attained. Try it first for air → glass then generally! ($n_t=1.50$)

Answer: a) $t^{\perp}=0 \Rightarrow \cos\theta_i=0 \Rightarrow \theta_i=90^{\circ}$

This is a trivial "glancing incidence" case

b) $n_i\cos\theta_i=n_t\cos\theta_t$

c) we need $n_i\sin\theta_i=n_t\sin\theta_t$.

air $n_i=1$

glass $n_t=1{,}50$ $\Rightarrow$ $\cos\theta_i=1{,}50\cos\theta_t$

$\sin\theta_i = 1.50\sin\theta_t$

Now $\cos^2\theta_i+\sin^2\theta_i=1 \Rightarrow (1.50)^2\underbrace{(\cos^2\theta_t+\sin^2\theta_t)}_{1}=1$

impossible.

In general $(n_i\cos\theta_i)^2+(n_i\sin\theta_i)^2=(n_t\cos\theta_t)^2+(n_t\sin\theta_t)^2$

$\Rightarrow n_i^2=n_t^2$

This is only possible when $n_i=n_t$. Then we need

$\sin\theta_i=\sin\theta_t \Rightarrow \theta_i=\theta_t \Rightarrow \cos\theta_i=\cos\theta_t$

(no reflection)

To assess the general behavior, note that

$$r^{\perp} = \frac{\frac{n_i}{n_t}\cos\theta_i - \cos\theta_t}{\frac{n_i}{n_t}\cos\theta_i + \cos\theta_t}$$

But $\cos\theta_t$ will be determined via Snell's law since

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t}$$

and $$\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i$$

Thus

$$r^{\perp} = \frac{\frac{n_i}{n_t}\cos\theta_i - \sqrt{1 - \left(\frac{n_i}{n_t}\right)^2 \sin^2\theta_i}}{\frac{n_i}{n_t}\cos\theta_i + \sqrt{1 - \left(\frac{n_i}{n_t}\right)^2 \sin^2\theta_i}}$$

or slightly less conventionally

$$r^{\perp} = \frac{\cos\theta_i - \sqrt{\left(\frac{n_t}{n_i}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_t}{n_i}\right)^2 - \sin^2\theta_i}}$$

We can plot this as a function of θ_i. To do this we need to consider two cases:

i) $n_i < n_t$ (light is incident in a faster medium)

ii) $n_i > n_t$ (light is " " " slower ")

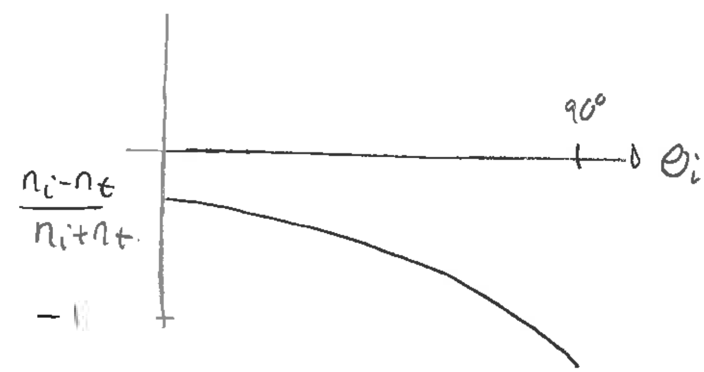
a) Light incident in a faster medium (s polarization) $(n_t > n_i)$

The quantity in the square root is always real Since $\cos\theta_i > 0$ and $\sqrt{} > 0$ we find that $r^\perp \leq 1$ in all cases. We use $\cos\theta_i = \sqrt{1-\sin^2\theta_i}$ to plot.

$$r^\perp = \frac{\sqrt{1-\sin^2\theta_i} - \sqrt{\left(\frac{n_t}{n_i}\right)^2 - \sin^2\theta_i}}{\sqrt{1-\sin^2\theta_i} + \sqrt{\left(\frac{n_t}{n_i}\right)^2 - \sin^2\theta_i}}$$

One can show that this is a decreasing function of θ_i. However it is negative. It attains a max at $\theta_i = 0$ and this is

$$\frac{1-\frac{n_t}{n_i}}{1+n_t/n_i}$$

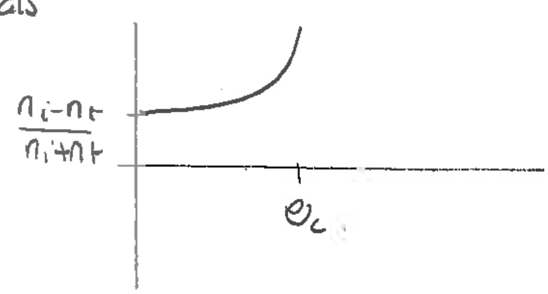


b) Light incident in a slower medium (s polarization) $(n_t < n_i)$

Here a similar rule applies provided $\frac{n_t}{n_i} > \sin\theta_i$. However this is an increasing function of θ_i.

We get $r^\perp = 1$ when θ_i equals the critical angle for total internal reflection

$$\theta_c = \sin^{-1}\left(\frac{n_t}{n_i}\right)$$



Parallel polarization

Exercise 2 Suppose that incident light is polarized parallel to the plane of incidence.
Using the Fresnel equations and Snell's Law determine a condition for $r^{\parallel} = 0$.

Answer $n_t \cos\theta_i = n_i \cos\theta_t$

$$n_i \sin\theta_i = n_t \sin\theta_t \Rightarrow n_t = n_i \frac{\sin\theta_i}{\sin\theta_t}$$

$$\Rightarrow n_t \cos\theta_i = \cancel{n_i} \frac{\sin\theta_i}{\sin\theta_t} \cos\theta_i = n_i \cos\theta_t$$

$$\Rightarrow \sin\theta_i \cos\theta_i = \sin\theta_t \cos\theta_t$$

$$\Rightarrow \sin(2\theta_i) = \sin(2\theta_t)$$

The first possibility might be $2\theta_i = 2\theta_t \Rightarrow \theta_i = \theta_t$ but if $n_i \neq n_t$ then Snell's law prohibits this. Now it is also possible that $2\theta_i = 2\theta_t + 2n\pi \Rightarrow \theta_i = \theta_t + n\pi$ and again the geometry prohibits this. Thus we try

$$2\theta_i = 2\theta_t + \pi$$

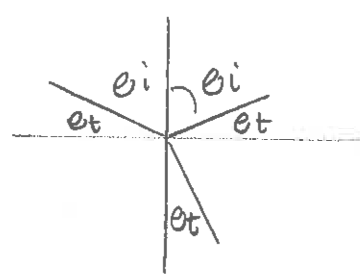
since $\sin(2\theta_t + \pi) = \sin(2\theta_t)\cos(\pi) + \ldots = -\sin 2\theta_t$. This also does not work. So $2\theta_i = -2\theta_t + \pi$ will give the correct result. Then $\theta_i + \theta_t = \pi/2 \Rightarrow \theta_t = -\theta_i + \pi/2)$

Now

$$n_t \cos\theta_i = n_i \cos\theta_t$$

$$= n_i \cos(-\theta_i + \pi/2)$$

$$= n_i \sin\theta_i \sin\pi/2$$



$$\Rightarrow \tan\theta_i = \frac{n_t}{n_i}$$

This is called Brewster's angle:

Brewster's angle is the angle at which no parallel polarized light is reflected. It satisfies:

$$\tan \theta_B = \frac{n_t}{n_i}$$