Phys 473 2015

Lecture 9

HW 6 Fri

HW 7 Weds

Reflection + Refraction

Previously we saw that the laws of reflection and refraction could be derived using the assumptions:

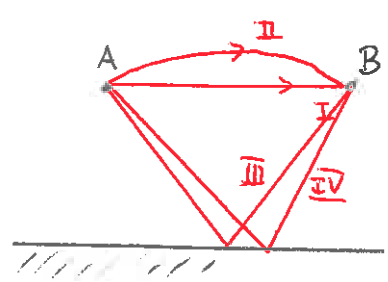
1) light is a wave
2) for light encountering an interface, the frequency of the transmitted and reflected light is unchanged from that of the incident light
3) there is a precise phase relationship between reflected or transmitted light waves and incident light waves.

These and various pieces of geometrical reasoning result in the laws of reflection + refraction. However, they both require an assumption that light is a wave.

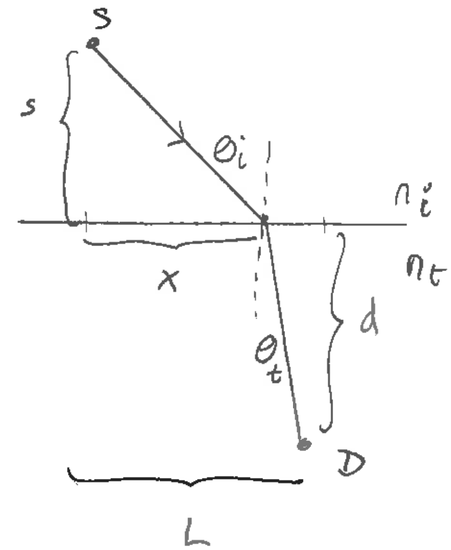
These laws can, however, be derived without resorting to waves. The key law in this case is:

> The path taken by light between any two points will be that which is such that the time taken for the light to travel between the points is a (local) minimum

This is Fermat's principle of least time. When applying it to reflection, for example, we choose two points as illustrated. Clearly a non-reflection path might take the shortest time. But a reflection path (III) might take a shorter time than any similar reflection path. This is the local minimum that we seek.



Exercise Apply Fermat's principle of least time to relate θ_i to θ_t for the illustrated refraction situation



Answer: The time taken is

$$\Delta t = \Delta t_{\text{incident medium}} + \Delta t_{\text{transmitted}}$$

$$= \frac{\Delta l_{\text{incident}}}{V_{\text{incident}}} + \frac{\Delta l_{\text{trans}}}{V_{\text{trans}}} \qquad V = c/n$$

$$= \frac{n_i\sqrt{s^2+x^2}}{c} + \frac{n_t\sqrt{d^2+(L-x)^2}}{c}$$

We need

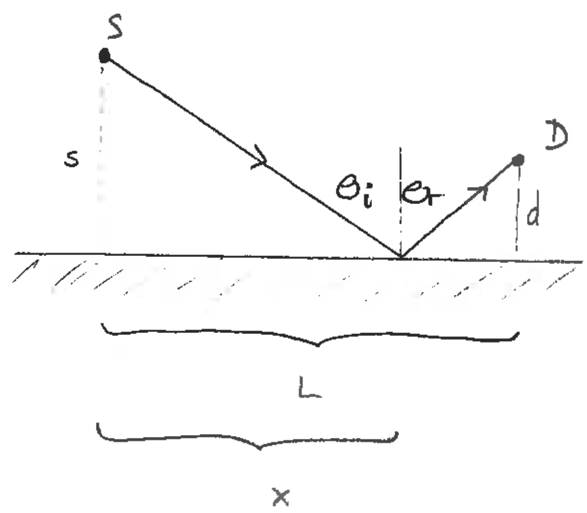
$$\frac{d\Delta t}{dx} = 0 \Rightarrow \frac{n_i x}{\sqrt{s^2+x^2}} - \frac{n_t(L-x)}{\sqrt{d^2+(L-x)^2}} = 0$$

$$\Rightarrow n_i \sin\theta_i - n_t \sin\theta_t = 0$$

$$\Rightarrow n_i \sin\theta_i = n_t \sin\theta_t$$

which gives Snell's Law

Exercise: Consider a source (S) and detector (D) point above a reflecting plane. These are fixed.



a) Determine an expression for the time taken for light to travel along the illustrated path, in terms of x, L, s, d.

b) Determine a condition for the light to take a minimum travel time.

c) Express this in terms of θ_i, θ_r.

Answer: a) $\Delta t = \Delta l / c$ where Δl is the distance traveled. Here

$$\Delta l = \sqrt{s^2+x^2} + \sqrt{d^2+(L-x)^2}$$

$$\Rightarrow \quad \Delta t = \frac{1}{c}\sqrt{s^2+x^2} + \frac{1}{c}\sqrt{d^2+(L-x)^2}$$

b) Need $\frac{d\Delta t}{dx} = 0 \Rightarrow \frac{x}{c\sqrt{s^2+x^2}} - \frac{(L-x)}{c\sqrt{d^2+(L-x)^2}} = 0$

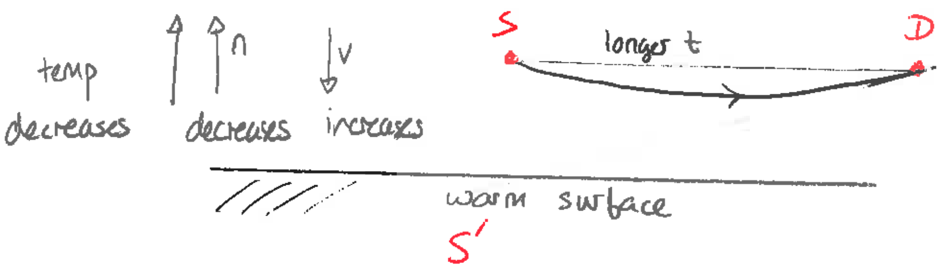
$$\Rightarrow \quad \frac{x}{\sqrt{s^2+x^2}} = \frac{L-x}{\sqrt{d^2+(L-x)^2}}$$

c) From geometry $\frac{x}{\sqrt{s^2+x^2}} = \sin\theta_i \qquad \frac{L-x}{\sqrt{d^2+(L-x)^2}} = \sin\theta_r$

$$\Rightarrow \quad \sin\theta_i = \sin\theta_r \quad \Rightarrow \quad \theta_i = \theta_r$$

Mirage formation

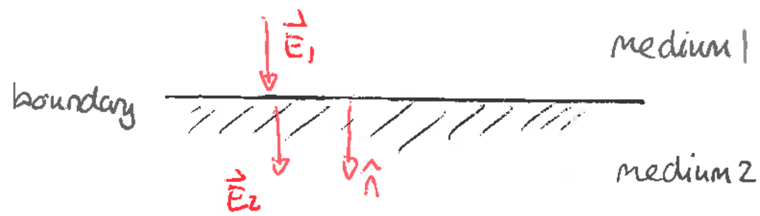
A mirage forms when light passes through air with a temperature gradient. In general as air temperature decreases, the index of refraction decreases and the speed of light increases. The shortest time path is no longer one which is a straight line but likely a curved path as illustrated



The direction at which the light ray approaches D is then not in a straight line from S. Thus the apparent direction of the light indicates an apparent source as illustrated. This produces images in the surface which are mirror-like.

Reflection + Refraction in terms of electromagnetic waves.

The starting point for using an electromagnetic wave description of reflection + refraction is to relate electric fields across the boundary between two uniform media



Consider medium 1 and medium 2. Let $\hat{n}$ be the normal vector to the surface pointing into medium 2. Then Maxwell's equations imply that

$$\vec{E}_2 - \vec{E}_1 = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$\vec{B}_2 - \vec{B}_1 = \mu_0 (\vec{K} \times \hat{n})$$

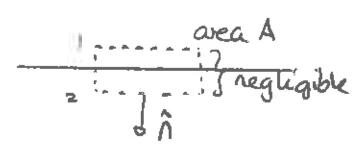
where σ is the surface charge density

$\vec{K}$ " " surface current density

Proof: We illustrate how the conditions for $\vec{E}$ arise. First consider

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \Rightarrow \oint_{\text{surface}} \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

and we consider the illustrated pillbox surface.



Then $Q_{enc} = \sigma A$

$$\oint \vec{E}\cdot d\vec{a} = \int_2 \vec{E}\cdot d\vec{a} - \int_1 \vec{E}\cdot d\vec{a} \Rightarrow \vec{E}_2\cdot\hat{n}\cancel{A} - \vec{E}_1\cdot\hat{n}\cancel{A} = \sigma\cancel{A}/\epsilon_0$$

$$\Rightarrow (\vec{E}_2 - \vec{E}_1)\cdot\vec{n} = \sigma/\epsilon_0$$

This describes the component perpendicular to the surface.

Now $\qquad \vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\vec{B} = \oint_{\text{loop}} \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$

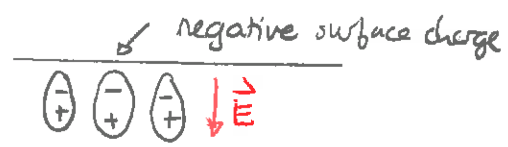
We consider various loops as illustrated.

As the height shrinks to zero $\int \vec{B}\cdot d\vec{a} \to 0$

But the l.h.s gives $\quad E_{1\,\text{tangential}} = E_{2\,\text{tangential}}$ and so the component tangential to the surface matches. Thus $\quad \vec{E}_2 - \vec{E}_1 = \sigma/\epsilon_0\,\hat{n}$

Similar derivations apply for the magnetic field

Unfortunately these require surface current + charge densities. The surface charge density might consist of free (excess) charges that have been placed on the surface as well as charges resulting from polarization in the presence of an external field. These effects can be described for linear dielectrics. In this case the net



electric field $\vec{E}$ is related to the electric displacement $\vec{D}$ by $\vec{D} = \epsilon\vec{E}$ where ϵ is the relative permittivity of the material. Applying classical electromagnetism gives

$$\vec{\nabla}\cdot\vec{D} = \rho_{\text{free}}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

where ρ_{free} only includes the free charge. The boundary conditions then become:

$$(\vec{D}_2 - \vec{D}_1)\cdot\hat{n} = \sigma_{\text{free}}$$

which describe the perpendicular field and

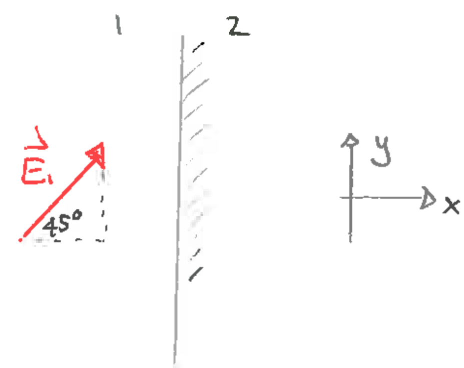
$$\vec{E}_{2\parallel} = \vec{E}_{1\parallel}$$

which describe any component of the field parallel to the interface

In optics situations we assume that there is no free surface charge. Thus

$$\boxed{\begin{aligned} (\epsilon_2 \vec{E}_2 - \epsilon_1 \vec{E}_1) \cdot \hat{n} &= 0 \\ \vec{E}_{2\parallel} &= \vec{E}_{1\parallel} \end{aligned}}$$

Exercise: Consider the illustrated surface and the indicated electric field vector $\vec{E}_1$



Suppose $\epsilon_2 > \epsilon_1$.

Qualitatively illustrated $\vec{E}_2$ and describe if E_{2y} is larger than or smaller than E_{2x}.

Answer: Clearly $E_{1y} = E_{1x}$

By the boundary conditions: $(\epsilon_2 \vec{E}_2 - \epsilon_1 \vec{E}_1) \cdot \hat{x} = 0$

$$\Rightarrow \epsilon_2 E_{2x} - \epsilon_1 E_{1x} = 0$$

$$\Rightarrow \quad E_{2x} = \frac{\epsilon_1}{\epsilon_2} E_{1x} < E_{1x} = E_{1y}$$

Also $\quad \vec{E}_{2\parallel} = \vec{E}_{1\parallel} \Rightarrow \quad E_{2y} = E_{1y}$

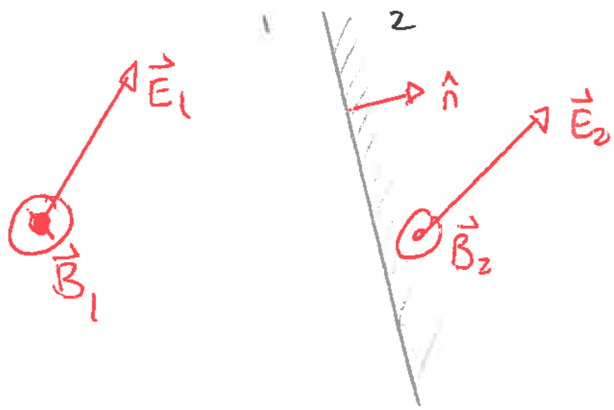
so $E_{2x} < E_{2y}$

Vector tilts.

Similar boundary conditions apply for magnetic fields. However, we will assume materials that are magnetically inactive and produce no surface currents. Thus we get

$$\boxed{\vec{B}_2 = \vec{B}_1}$$

To summarize:



$$(\epsilon_2 \vec{E}_2 - \epsilon_1 \vec{E}_1) \cdot \hat{n} = 0$$

$$\vec{E}_{2\parallel} = \vec{E}_{1\parallel}$$

$$\vec{B}_2 = \vec{B}_1$$