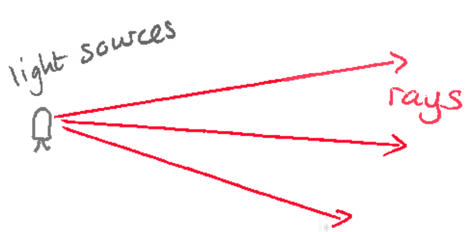
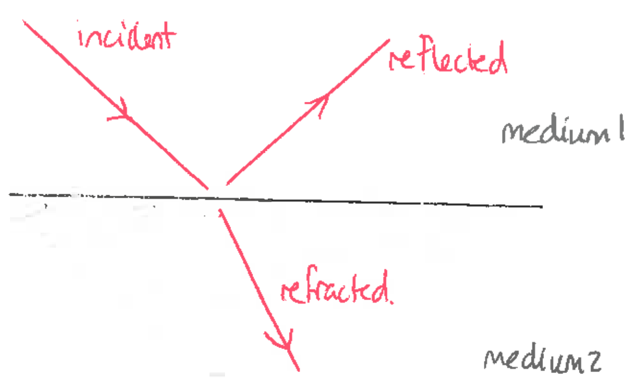
HW 6 due by Friday - note extra problems.

Reflection + Refraction

One method, used for many centuries, for understanding certain optical phenomena describes light in terms of rays. The exact nature of these rays is unspecified but rules regarding the way in which they propagate or travel through space are provided. These rules can be used to explain phenomena such as reflection + refraction or the workings of lenses and other optical devices. In a uniform homogeneous + isotropic medium, the rays are assumed to propagate in straight lines. When they pass from one medium to another with different properties, the rays change direction. These changes and their implications can often be analyzed using straight line geometry. Hence this variant of the study of light is called geometric optics (sometimes ray optics)



Two of the key phenomena associated with this occur when light traveling through one uniform medium encounters another medium. There are two possibilities



1) some light is reflected back

2) some light is transmitted and may change direction. This is refraction

We aim to:

1) provide geometric rules that the various light rays obey in both processes

2) using electromagnetism and electromagnetic waves to describe how these processes unfold.

These phenomena both have to do with the change in speed of light from one medium to another. Recall that the index of refraction describes speed in a medium via

$$\boxed{n = \frac{\text{speed of light in vacuum}}{\text{speed in medium}} = \frac{c}{v}}$$

The index of refraction depends mostly on the medium but also partly on the frequency of the incident light. (see Table 3.1)

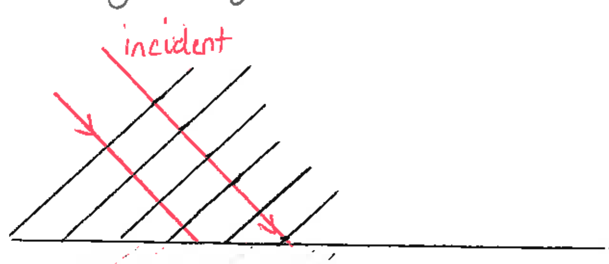
One way to assess both of these phenomena uses the facts that

i) $v = \lambda f$

ii) the frequency of light does not change when passing from one medium to another

Law of Reflection

Consider a light ray that is reflected at a surface. Considering that the light is a wave we can assume a plane wave and illustrate several crests that are perpendicular to the incident ray.

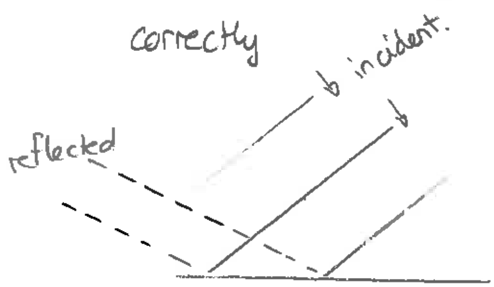


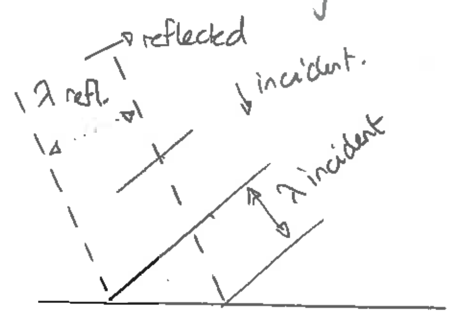
The key properties of the reflected wave are:

i) its wavelength is the same as that of the incident wave since the wave speed is the same.

ii) whereever a crest of the incident wave encounters the interface, there must also be a trough of the reflected wave.

Then there is only one way to construct the troughs of the reflected wave correctly





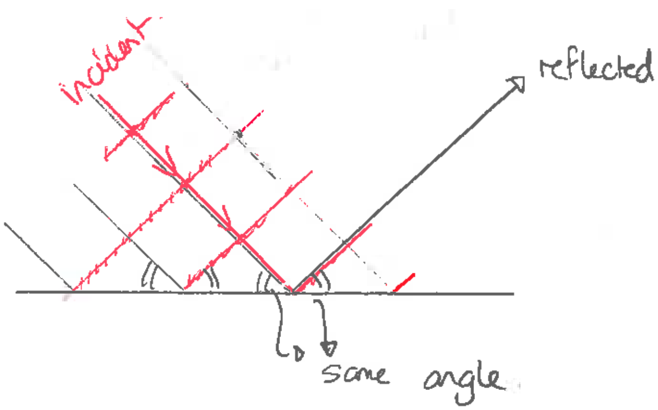
reflected troughs indicate smaller wavelength

$\Rightarrow$ incorrect

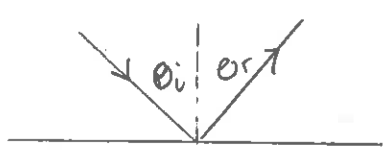
reflected troughs, indicate larger wavelength

$\Rightarrow$ incorrect

The only way to satisfy both is that the troughs of the reflected wave make the same angle with the surface as do those of the incident wave



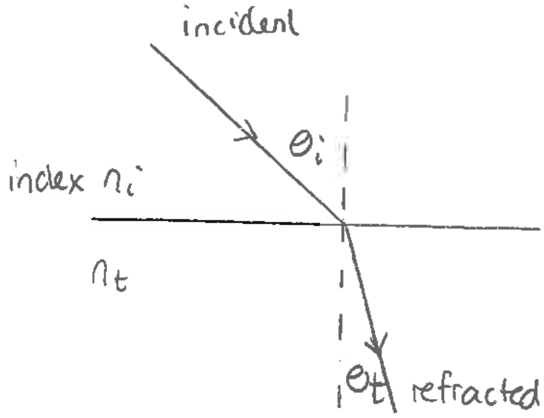
Removing the crests gives the law of reflection:



Using the indicated angles,

$$\theta_i = \theta_r$$

A light ray that is transmitted from one medium to another typically emerges along a different direction. This is called <u>refraction</u>. The relevant angles used to describe this are illustrated. These are the angles from the normal to the surface and each ray. Then <u>Snell's Law</u> states that:



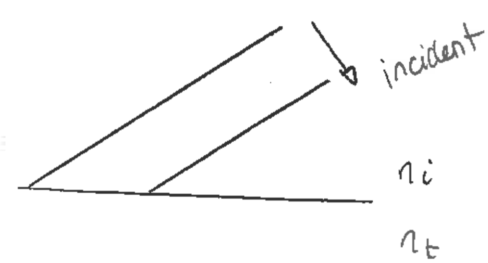
> If the index of refraction of the incident ray medium is n_i and n_t for the transmitted ray medium, then
>
> $$n_i \sin\theta_i = n_t \sin\theta_t$$

The following exercise provides one derivation of this:

<u>Exercise</u>: a) Determine a relationship between the wavelength of the light in the "incident" medium and that in the "transmitted" medium, in terms of indices of refraction.

b) Suppose that $n_t > n_i$. Is the wavelength in the transmitted medium, λ_t, larger or smaller than that in the incident medium, λ_i?

c) Two adjacent crests for the wave in the incident medium are provided. Again suppose $n_t > n_r$. On the diagram:



i) indicate λ_i, θ_i

ii) draw the corresponding crests in the transmitted ray medium.

iii) indicate λ_t, θ_t

Then relate these to derive Snell's Law.

Answers: a) The frequencies are the same so $f = V/\lambda$ is same.

Thus

$$\frac{V_t}{\lambda_t} = \frac{V_i}{\lambda_i}$$

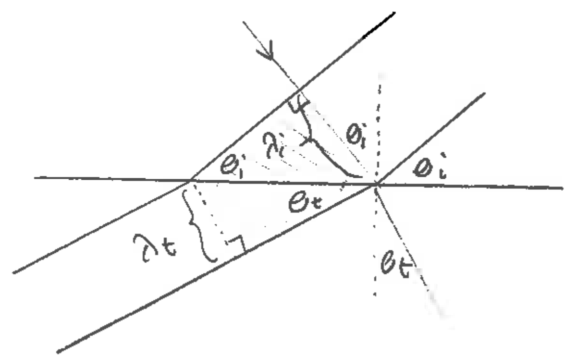
But $n_t = c/V_t \Rightarrow V_t = c/n_t$ etc,...

Thus $$\frac{c}{\lambda_t n_t} = \frac{c}{\lambda_i n_i} \Rightarrow \lambda_t n_t = \lambda_i n_i$$

$$\Rightarrow \boxed{\lambda_t = \lambda_i \frac{n_i}{n_t}}$$

b) smaller by part a)

c)



The shaded triangles share a common base. Denote the length of this by x. Then trigonometry gives:

$$\frac{\lambda_t}{x} = \sin\theta_t \qquad \frac{\lambda_i}{x} = \sin\theta_i$$

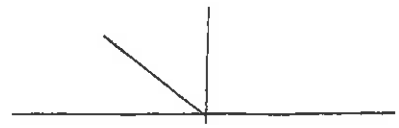
$$\Rightarrow x = \frac{\lambda_t}{\sin\theta_t} \qquad x = \frac{\lambda_i}{\sin\theta_i}$$

$$\Rightarrow \frac{\lambda_t}{\sin\theta_t} = \frac{\lambda_i}{\sin\theta_i} \Rightarrow \lambda_i \sin\theta_t = \lambda_t \sin\theta_i$$

$$\Rightarrow \lambda_i \sin\theta_t = \lambda_i \frac{n_i}{n_t} \sin\theta_i$$

$$\Rightarrow n_t \sin\theta_t = n_i \sin\theta_i$$

Total Internal Reflection.



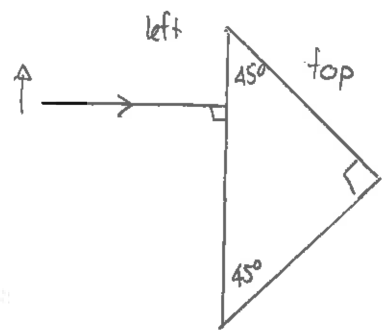
In general

$$\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i$$

and so when $n_i > n_t$ it is possible for the l.h.s to be greater than 1. Whenever this occurs no light is transmitted. This is total internal reflection. The limiting angle of incidence occurs when $\theta_t = 90°$ and the corresponding limiting angle of incidence is the critical angle, θ_c, where

$$1 = \frac{n_i}{n_t}\sin\theta_c \Rightarrow \sin\theta_c = n_t/n_i \Rightarrow \theta_c = \sin^{-1}\left(\frac{n_t}{n_i}\right)$$

__Exercise__ Light is incident on the illustrated prism.



a) Determine the path of the light through the left face.

b) Suppose that total internal reflection occurs at the "top" surface. Trace the path of the light through the remaining faces.

Will this prism produce inverted or upright images?

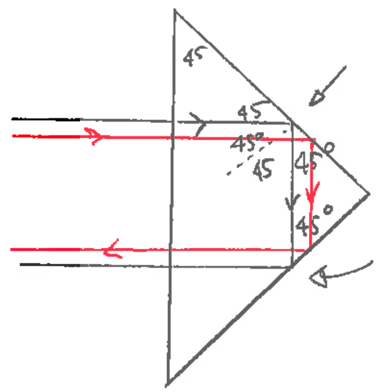
c) Determine the minimum index of refraction so that total internal reflection occurs on the "top" surface. (when the external medium is air)

Answers: a) At left surface $\theta_i = 0°$

$$n_i \sin\theta_i = n_t \sin\theta_t \Rightarrow 0 = n_t \sin\theta_t$$

$$\Rightarrow \theta_t = 0°$$

b)



angle of reflected ray equals angle of incident ray

same here

The two rays indicate inversion of an image.

c) We need that $45° > \theta_c$

~~glass~~ air

$$\Rightarrow 45° > \sin^{-1}\left(\frac{n_t}{n_i}\right)$$

glass

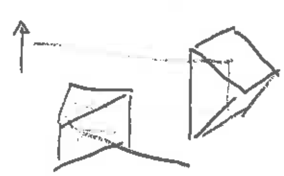
$$\Rightarrow \sin 45° > n_t/n_i$$

$$\Rightarrow \frac{1}{\sqrt{2}} > \frac{n_t}{n_i} \Rightarrow \frac{n_i}{n_t} > \sqrt{2}$$

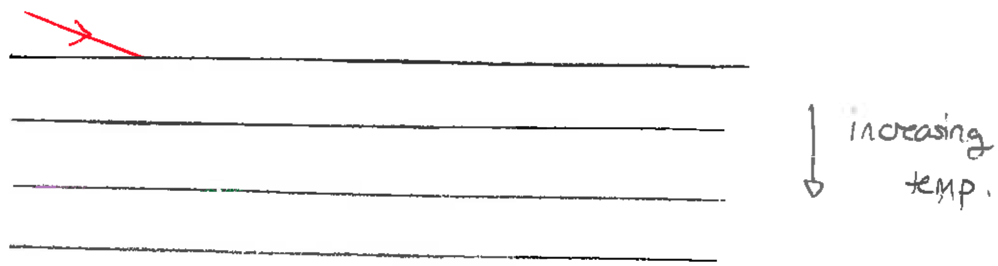
$$\Rightarrow n_{glass} > n_{air}\sqrt{2}$$

$$\Rightarrow n_{glass} > \sqrt{2}$$

Such prisms are used in pairs to re-invert images produced by inverting lenses

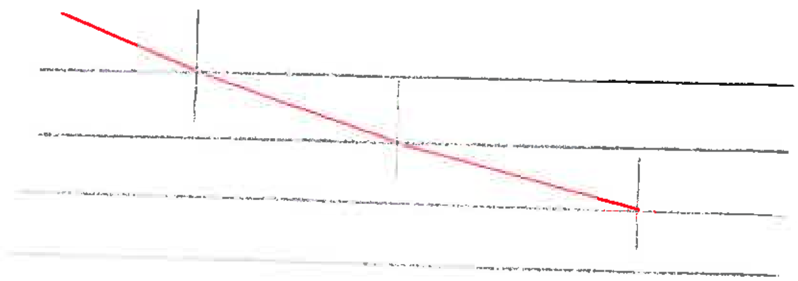


Exercise 3: Warmer air has a lower index of refraction that colder air. Consider several layers of air.



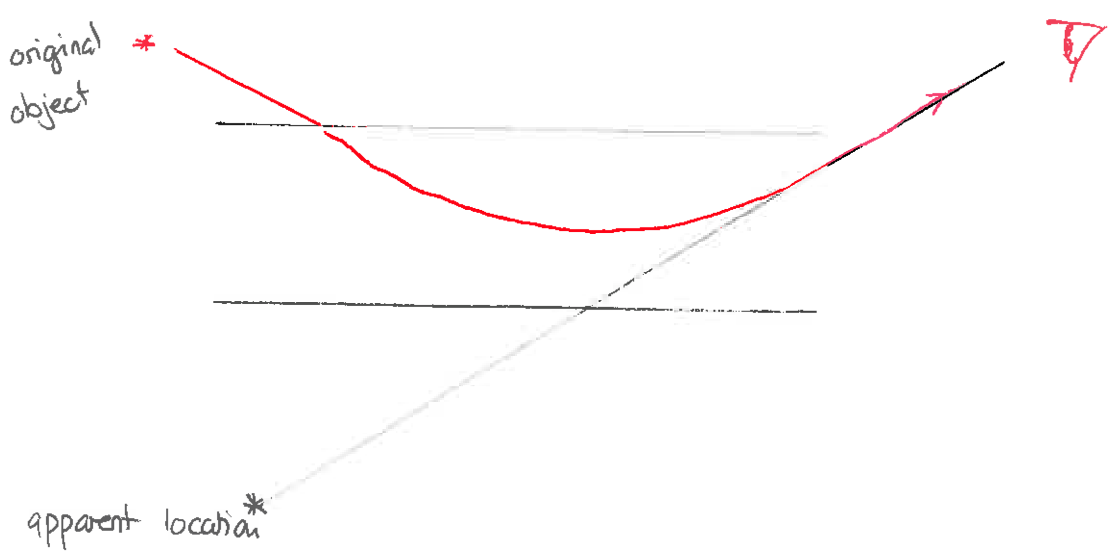
Trace an approximate path for the illustrated incident ray.

Answer:



If $n_t < n_i$ then $\theta_t > \theta_i$ until eventually internal reflection occurs □

We see the path is:



This explains how mirages appear.