Lecture 1

Intro: - Syllabus

- Course structure - meetings
- Assignments - HW typically M,F
  - Two class exams, ~~one~~
  - One Final exam
- Grade structure - see syllabus.

Scope of Optics

Optics is the science of light and how to manipulate light. Observation of everyday visual phenomena has long motivated the search for explanations of phenomena such as:

- reflection
- distortion of light passing through water
- rainbows
- irridesence, etc, ..

Also, it has long been clear that light can be manipulated via lenses and mirrors. Indeed, as early as the 17th century, Galileo had constructed a telescope and used it to observe astronomical objects. The resulting observations yielded evidence that strongly motivated a change in the understanding of the physical universe.

Issues like these have provided practical motivation for developing the science of optics for hundreds of years.

But there has also long been a larger fundamental question: What exactly is light?

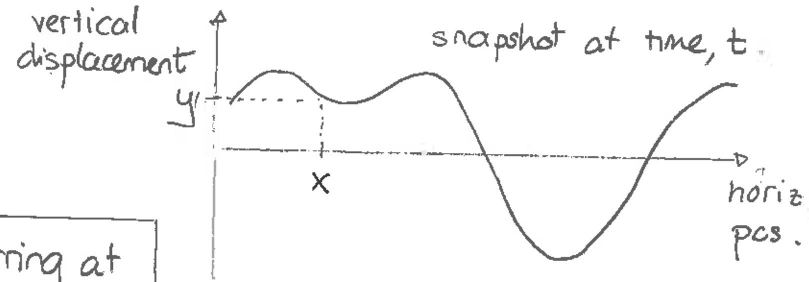
The first convincing comprehensive answer to this emerged in the 19th century once a complete theory of classical electromagnetism was developed. Merging this with the theory of waves resulted in a classical theory of optics that was compact and satisfactorily explained most known optical phenomena.

This theory, it emerged is not complete and a more modern merger of electromagnetism and quantum theory has yielded quantum optics which successfully explains certain phenomena associated with low light intensities.

This course primarily describes the classical theory of optics. You will see how one can combine two theories from physics to produce a powerful new offshoot. You will also learn about many phenomena of optics and how to explain these. Separately this opens the door to understanding optical instruments, devices + sensing techniques that are widespread throughout sciences. Some of our most precise measurements rely on optical sensing.

Wave equation

If one considers the transverse motion of a stretched string, we aim to describe the vertical displacement of the string as a function of horizontal position and time. This is described by a displacement function



$$\boxed{y(x,t) = \text{displacement of string at location } x \text{ at time } t.}$$

Classically we arrive at an equation of motion for this by applying Newton's Laws to segments of the string. This results in the fact that the displacement function satisfies a differential equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}$$

where μ is the mass per unit length of the string and T is the tension in the string. This is an example of the one-dimensional wave equation for a function $\Psi(x,t)$:

$$\boxed{\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 \Psi}{\partial t^2}}$$

where v is a real constant (i.e. independent of x and t). Knowing the solution to this equation, i.e. $\Psi(x,t)$ allows one to completely describe the relevant physical situation. A solution to this means:

> A function $\Psi(x,t)$ that, when substituted into the wave equation, gives an algebraic expression that is true for all independent choices of x and t.

Exercise: Consider the function

$$\Psi(x,t) = B e^{-(x-vt)^2/a^2}$$

where a has units of position.

a) Show that this is a solution to the wave equation

b) Plot this at $t = 0s$. Where is the maximum located?

c) Where is the maximum at $t = t'$? With what speed has the maximum traveled?

Answer: a)

$$\frac{\partial \Psi}{\partial x} = B e^{-(x-vt)^2/a^2} \frac{\partial}{\partial x} \left\{ -(x-vt)^2/a^2 \right\}$$

$$= B e^{-(x-vt)^2/a^2} \left(-\frac{2}{a^2}\right)(x-vt)$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{2B}{a^2} \frac{\partial}{\partial x} \left\{ e^{-(x-vt)^2/a^2} (x-vt) \right\}$$

$$= -\frac{2B}{a^2} \left[e^{-(x-vt)^2/a^2} \left(-\frac{2}{a^2}\right)(x-vt)(x-vt) + e^{-(x-vt)^2/a^2} \right]$$

$$= \frac{-2B}{a^2} e^{-(x-vt)^2/a^2} \left[\left(-\frac{2}{a^2}\right)(x-vt)^2 + 1 \right]$$

$$\frac{\partial \Psi}{\partial t} = \cdots = B e^{-(x-vt)^2/a^2} \left(-\frac{2}{a^2}\right)(-v)(x-vt)$$

$$\frac{\partial^2 \Psi}{\partial t^2} = \cdots = \frac{-2B}{a^2} e^{-(x-vt)^2/a^2} \left[v^2 (x-vt)^2 \left(-\frac{2}{a^2}\right) + v^2 \right]$$

$$= \frac{-2B}{a^2} e^{-(x-vt)^2/a^2} \left[-\frac{2}{a^2}(x-vt)^2 + 1 \right] v^2$$

We see that $\frac{\partial^2 \Psi}{\partial t^2} = v^2 \frac{\partial^2 \Psi}{\partial x^2}$ and this satisfies the wave equation.

b) At $t = 0$ $\Psi(x,0) = Be^{-x^2/a^2}$ has a max at $x = 0$

c) At $t = t'$ $\Psi(x,t') = Be^{-(x-vt')^2/a^2}$ has a max when $x - vt' = 0$

$\Rightarrow x = vt'$.

The speed with which the max traveled is $\frac{\Delta x}{\Delta t} = \frac{vt' - 0}{t' - 0} = v$.

We see that in this case the constant v represents the speed with which the wave travels.

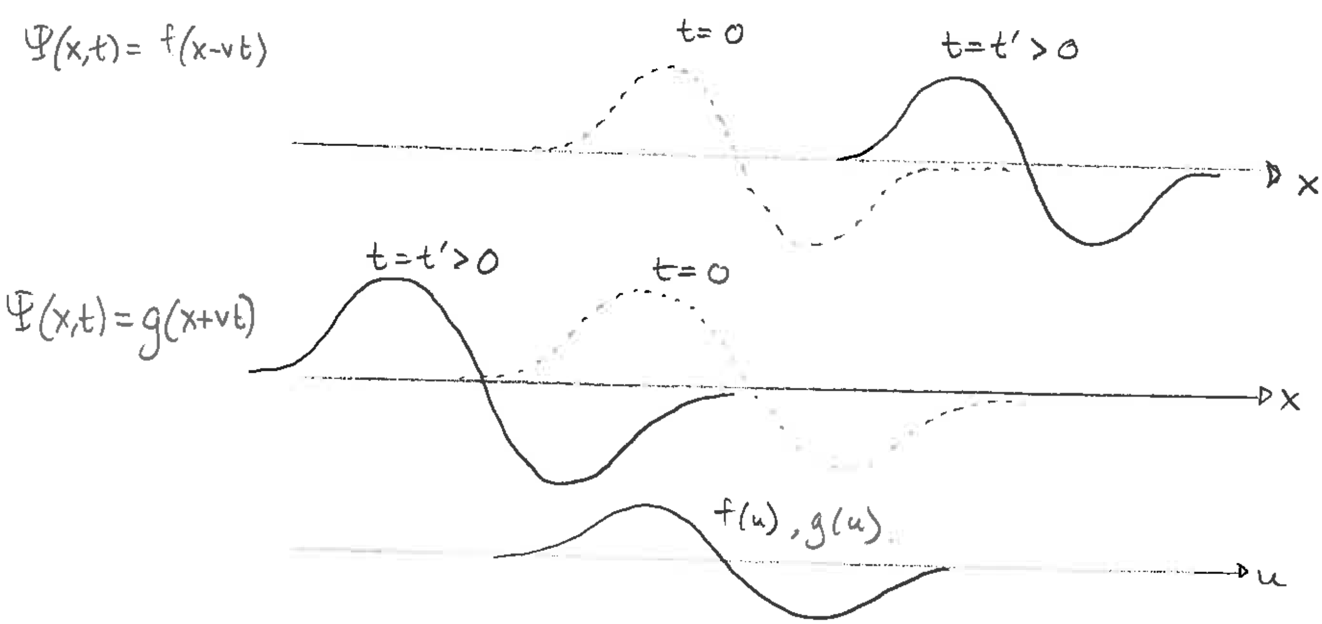
General solution

There are many possible solutions to the wave equation but they are captured in all generality as:

> The most general solution to the wave equation is:
>
> $$\Psi(x,t) = f(x-vt) + g(x+vt)$$
>
> where f and g are any twice differentiable functions.

Considering the two parts separately we can easily show that

> $f(x-vt)$ represents a wave traveling to the right with speed v
>
> $g(x+vt)$ " " " " " " left " " v



Harmonic solutions.

Harmonic solutions are those for which f, g are sine and cosine functions. Thus, with ϕ constant,

$$f(u) = A\sin(ku+\phi) \Rightarrow \Psi(x,t) = f(x-vt+\phi$$

$$\Rightarrow \Psi(x,t) = A\sin(kx-kvt+\phi)$$

is a solution that propagates to the right. The constant k is called the wavenumber. The angular frequency is defined as:

$$\boxed{\omega := kv}$$

and this gives a general right traveling harmonic solution:

$$\boxed{\Psi(x,t) = A\sin(kx-\omega t+\phi)}$$

The general left traveling harmonic solution is:

$$\boxed{\Psi(x,t) = A\sin(kx+\omega t+\phi)}$$

In both cases the constants ϕ are called phase constants. The wavelength of the wave is:

$$\boxed{\lambda := \frac{2\pi}{k}}$$

and the frequency

$$\boxed{f := \omega/2\pi}$$

It follows that

$$\boxed{\lambda f = v}$$