Modern Optics: Homework 25

Due: 2 December 2015

1 Saturation of Two-Level Atoms

Consider two non-degenerate levels of an atom. With the usual notation, the populations of the levels satisfy:

$$\frac{dN_2}{dt} = -N_2 A_{21} + B_{12} N_1 u(f) - B_{21} N_2 u(f)$$

where $u(f)$ is the spectral energy density function. Denote $A_{21} = A$ and using the result that $B_{21} = B_{21}$, let $B := B_{21} = B_{21}$.

a) Substitute $N_1 = N - N_2$ into the equation above and assume a solution of the form

$$N_2(t) = \alpha e^{\beta t} + \gamma$$

where α, β, and γ are constants. Verify that this does solve the differential equation for N_2 and determine expressions for β and γ in terms of A, B, N and $u(f)$.

b) Use this to show that

$$N_2(t) = \left[N_2(0) - N \frac{Bu(f)}{A + 2Bu(f)} \right] e^{-(A+2Bu(f))t} + N \frac{Bu(f)}{A + 2Bu(f)}.$$

c) Show that as $t \to \infty$, we obtain the equilibrium solution for $u(f)$ as described in class.

d) Determine equilibrium expressions for N_2/N and N_1/N.

2 Bennett, *Principles of Physical Optics*, 7.10, page 360.