Phys 473
Fall 2015

Modern Optics: Homework 19

Due: 28 October 2015

1 Bennett, *Principles of Physical Optics*, 5.26, page 234.

2 Bennett, *Principles of Physical Optics*, 5.27, page 234.

3 Waves in a lattice

In a one dimensional lattice (of masses or atoms) the dispersion relation is

$$\omega(k) = \omega_0 \sin\left(\frac{kl}{2}\right)$$

where l is the spacing between lattice sites and ω_0 is a constant

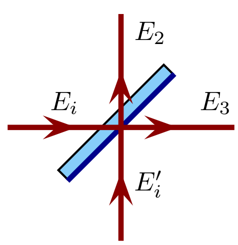
a) Determine an expression for the phase velocity in this lattice.

b) As the wavelength of a harmonic wave increases, does the phase velocity increase or decrease?

c) Determine an expression for the group velocity in this lattice.

4 Beam Splitter Relations

Consider the illustrated beam splitter, with beams incident from the bottom and left.



Then

$$E_2 = rE_i + t'E_i'$$
$$E_3 = tE_i + r'E_i'.$$

a) Using energy conservation, show that setting $E_i' = 0$ requires $|r|^2 + |t|^2 = 1$.

b) Using energy conservation, show that setting $E_i = 0$ requires $|r'|^2 + |t'|^2 = 1$.

c) In general neither of input field is zero. Consider the general case and use energy conservation to show that

$$\left(tr'^* + rt'^*\right) E_i E_i'^* + \left(t^* r' + r^* t'\right) E_i^* E_i' = 0.$$

d) The only way that these can be true for any input fields is if

$$tr'^* + rt'^* = 0$$

and

$$t^* r' + r^* t' = 0.$$

The latter is just the complex conjugate of the first. Use these to show that

$$\frac{|r|}{|t|} = \frac{|r'|}{|t'|}$$

and use these results to show that

$$|r| = |r'|$$
$$|t| = |t'|.$$

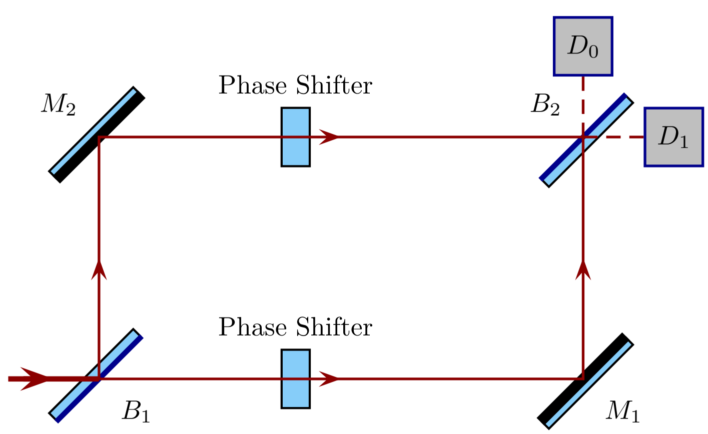
e) Use the preceeding results to show that

$$\phi_t - \phi_r' = \phi_r - \phi_t' \pm \pi.$$

This is a corrected version of the expression in class.

5 Mach-Zehnder Interferometer

Consider the illustrated Mach-Zehnder interferometer with two different phase shifters inserted into the two arms. These generate different phases ϕ_A and ϕ_B along the two paths.



a) Determine general expressions for the fields at the two detectors in terms of the reflection and transmission coefficients for the beam splitters (do not assume that they are 50-50 beam splitters).

b) Determine expressions for the intensities at the detectors.

c) Suppose that the beam splitters are 50-50. Determine expressions for the intensities at the detectors. Show that total energy is conserved.

d) Suppose that the phase shifters are identical and the path lengths are identical but that the beam splitters are not necessarily 50-50. Determine expressions for the intensities at the detectors. Show that total energy is conserved.