Modern Optics: Homework 18

Due: 23 October 2015

1 Bennett, *Principles of Physical Optics*, 5.18, page 226.

2 Wavepackets

Consider a wavepacket described at $t = 0$, by

$$\psi(x) = \frac{A}{b^2 + x^2}\, e^{ik_0 x}$$

where $b > 0$ and $k_0 > 0$.

a) Using full-width at half maximum (FWHM) as a measure of the spatial spread of this wave, Δx, determine Δx. As b increases does the spatial spread increase or decrease?

b) Determine the distribution of wavenumbers $\tilde{\psi}(k)$ and determine the spread in these, Δk using the FWHM. As b increases does the wavenumber range increase or decrease?

c) Verify that $\Delta k \Delta x$ does not depend on b.

Hint: If $a > 0$ then

$$\int_{-\infty}^{\infty} \frac{e^{ikx}}{b^2 + x^2} = \frac{\pi e^{-|k|b}}{b}.$$

3 Pulsed nuclear magnetic resonance

Nuclear magnetic resonance uses the fact that the magnetic dipole moments of nuclei oscillate between two states when placed in a strong constant magnetic field. This characteristic frequency of oscillation depends on the strength of the external magnetic field and the local chemical environment of the various nuclei. The state of any one nuclear dipole can be manipulated by applying an additional weak oscillating external magnetic field. The effect of this additional magnetic field is most pronounced when its frequency matches the characteristic frequency of the nucleus; this is similar to resonance. In practice the additional field is supplied by an oscillating pulse with a finite duration Δt and this limits its ability to selectively manipulate just one nucleus while leaving the rest unaffected.

a) Suppose that a molecule contains just two nuclei, H and $^{13}\mathsf{C}$. In a particular magnetic field these oscillate with frequencies $f_{\mathsf{H}} = 500\,\text{MHz}$ and $f_{\mathsf{C}} = 125\,\text{MHz}$. Estimate the minimum duration of a pulse which should only affect the H nucleus.

b) The alanine molecule contains three distinct $^{13}\mathsf{C}$ nuclei which have slightly different frequencies as a result of their local chemical environments. In a particular magnetic field these are of the form $f = 125\times10^6\,\mathrm{Hz}+\delta f$ where δf are $0\,\mathrm{Hz}, 10160\,\mathrm{Hz}$ and $15830\,\mathrm{Hz}$ for each of the three nuclei. Estimate the minimum duration of the pulse which should only affect the nucleus with the middle frequency.