Phys 473
Fall 2015

Modern Optics: Homework 10

Due: 18 September 2015

1 Bennett, *Principles of Physical Optics*, 3.23, page 94.

2 Bennett, *Principles of Physical Optics*, 3.25, page 94.

3 Reflectivity and transmittivity for general polarization

We have developed rules for reflectivity and transmittivity for plane waves that are polarized perpendicular to the plane of incidence. However, the general incident wave could be polarized in any direction and the rules might be modified for this case. In general the electric field is $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ where $\mathbf{E}_0$ lies along any direction perpendicular to $\mathbf{k}$. This means that it can still be written as

$$\mathbf{E}_0 = \mathbf{E}_0^{\perp} + \mathbf{E}_0^{\parallel}$$

where the two vectors in this decomposition are perpendicular and parallel to the plane of incidence. The boundary conditions apply to each of these separately as before, i.e. $E_{ot}^{\perp} = t^{\perp} E_{oi}^{\perp}$, where $t^{\perp}$ is as before, etc, ...

a) Show that the irradiance associated with this wave is

$$I = I^{\perp} + I^{\parallel}$$

where $I^{\perp}$ is the irradiance associated with the perpendicular component alone and $I^{\parallel}$ is the irradiance associated with the parallel component.

b) Show that the reflectivity is

$$R = \frac{r^{\perp 2}\left(E_{0i}^{\perp}\right)^2 + r^{\parallel 2}\left(E_{0i}^{\parallel}\right)^2}{\left(E_{0i}^{\perp}\right)^2 + \left(E_{0i}^{\parallel}\right)^2}$$

and that the transmittivity is

$$T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\,\frac{t^{\perp 2}\left(E_{0i}^{\perp}\right)^2 + t^{\parallel 2}\left(E_{0i}^{\parallel}\right)^2}{\left(E_{0i}^{\perp}\right)^2 + \left(E_{0i}^{\parallel}\right)^2}.$$

c) By first showing that

$$\frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^{\perp 2} + r^{\perp 2} = 1$$

with a similar result for the parallel polarization, then show that

$$R + T = 1$$

for this general case.