# Modern Optics: Homework 8 

Due: 11 September 2015

## 1 Fresnel amplitude ratios

Using Snell's law show that the Fresnel amplitude ratios can be rewritten as

$$
\begin{aligned}
r^{\perp} & =\frac{\sin \left(\theta_{t}-\theta_{i}\right)}{\sin \left(\theta_{t}+\theta_{i}\right)} \\
t^{\perp} & =\frac{2 \sin \theta_{t} \cos \theta_{i}}{\sin \left(\theta_{t}+\theta_{i}\right)} \\
r^{\|} & =\frac{\tan \left(\theta_{t}-\theta_{i}\right)}{\tan \left(\theta_{t}+\theta_{i}\right)} \\
t^{\|} & =\frac{2 \sin \theta_{t} \cos \theta_{i}}{\sin \left(\theta_{t}+\theta_{i}\right) \cos \left(\theta_{t}-\theta_{i}\right)} .
\end{aligned}
$$

Note that to prove the last two you could first prove that

$$
\begin{aligned}
& \cos A \sin A+\cos B \sin B=\sin (A+B) \cos (A-B) \\
& \cos A \sin A-\cos B \sin B=\sin (A-B) \cos (A+B)
\end{aligned}
$$

## 2 Eliminating reflections

Suppose that you would like to eliminate the reflected light, which produces a glare, off a slanted surface. Assume that the reflected light travels horizontally toward your eye and that the surface is angled at $30^{\circ}$ from the horizontal.
a) Sketch a view from the side, indicating the reflected and incident light rays and determine $\theta_{i}$.
b) Suppose that the light is incident in air and is reflected off glass whose index of refraction is 1.50. Determine the Fresnel amplitude ratios for reflection for both polarizations. Use the results to describe how you would orient a polarizing filter (only lets one component of the electric field pass) so that most reflected light will be eliminated.
c) Show that for any material, with the incident light ray at this angle,

$$
\begin{aligned}
r^{\perp} & =\frac{1-\sqrt{3} \cos \theta_{t} / \sin \theta_{t}}{1+\sqrt{3} \cos \theta_{t} / \sin \theta_{t}} \\
r^{\|} & =\frac{2 \sin \left(2 \theta_{t}\right) / \sqrt{3}-1}{2 \sin \left(2 \theta_{t}\right) / \sqrt{3}+1}
\end{aligned}
$$

d) Verify that these yield the correct results for reflection off glass.
e) Suppose that light is reflected off water (index 1.33) at the same angle. Determine the reflection coefficients and describe in which direction to orient the polarizing filter so as to eliminate the most reflected light. Is the reduction in reflected light as pronounced as that for reflection off glass?

3 Bennett, Principles of Physical Optics, 3.8, page 76.

