Phys 473
Fall 2015

Modern Optics: Homework 7

Due: 9 September 2015

1 Mirages

Consider light traveling left to right along a path that may curve. The path can be written as $z = z(x)$. Suppose that the index of refraction depends on z, i.e. $n = n(z)$.

a) Show that the time taken for the light to travel along the path is

$$t = \frac{1}{c} \int n(z)\sqrt{1 + z'^2}\,\mathrm{d}x$$

where $z' := \dfrac{dz}{dx}$.

b) The calculus of variations states that the expression

$$\int F(x, z, z')\mathrm{d}x$$

is minimized when

$$\frac{d}{dx}\left(\frac{\partial F}{\partial z'}\right) = \frac{\partial F}{\partial z}.$$

Applying this to the problem of minimizing the time to show that

$$n(z)z'' - n'\left(1 + z'^2\right) = 0.$$

c) Show that this implies that the path followed by the light is concave up.

d) Use the previous result to show that

$$\frac{\sqrt{1 + z'^2}}{n} = \alpha$$

where α is constant and solve this for z'. This gives an equation for the path followed by the light.

e) Determining the exact path requires knowing $n(z)$. To check suppose that the index of refraction is constant. Determine an expression for $z(x)$ assuming that the source and detector of the light are at the same height above the ground. Does this yield a correct path?

f) *Extra credit (up to 20% of the total points for the entire assignment):* Suppose that $n(z) = n_0(1 - \beta z)$ where $\beta > 0$ is constant. Determine an expression for the path followed by the light from a source and detector, each at height h above the ground and separated by distance L horizontally.

2 Transmission at a surface

Two media are separated by a surface that lies in the $z = 0$ plane. For $z > 0$ the relative permittivity of the material is ϵ_1 and for $z < 0$ it is ϵ_2. An electromagnetic wave approaches this surface while traveling in the direction $\frac{1}{\sqrt{2}}\left(\hat{\mathbf{i}} - \hat{\mathbf{k}}\right)$. The magnetic field vector is oriented along the $\hat{\mathbf{j}}$ direction.

a) Determine the direction of the incident electric field vector.

b) Determine an expression (in terms of the permittivities) for the direction of propagation of the transmitted electromagnetic wave. Verify that this satisfies Snell's law.

c) Determine an expression for the ratio of the intensity of the transmitted to the incident wave.

d) Repeat all of these for the situation where the incident electromagnetic wave travels along the direction $\frac{1}{\sqrt{17}}\left(\hat{\mathbf{i}} - 4\hat{\mathbf{k}}\right)$

3 Bennett, *Principles of Physical Optics*, 3.4, page 75.

4 Bennett, *Principles of Physical Optics*, 3.5, page 75.