Modern Optics: Homework 5

Due: 31 August 2015

1 Bennett, *Principles of Physical Optics*, 2.8, page 38.

2 Bennett, *Principles of Physical Optics*, 2.9, page 38.

3 Direction of propagation versus electric and magnetic field orientations

Starting with $\mathbf{k} \times \mathbf{E} = \omega \mathbf{B}$ and using a vector algebra triple product identity, show that

$$\mathbf{E} \times \mathbf{B} = \frac{E^2}{\omega}\,\mathbf{k}$$

where $E^2 := \mathbf{E} \cdot \mathbf{E}$.

4 Energy and harmonic waves

Consider the harmonic electromagnetic waves described by

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0\ \cos{(kx - \omega t)}.$$

a) Determine the energy contained in the rectangular region $0 \leqslant x \leqslant \lambda/4, 0 \leqslant y \leqslant L_1$, and $0 \leqslant z \leqslant L_2$ where λ is the wavelength of the wave and L_1 and L_2 are constants with units of length. Simplify the resulting expression as much as possible (you should be able to reduce all the trigonometric functions that appear to just one).

b) Does the energy contained in the region $0 \leqslant x \leqslant \lambda/4, 0 \leqslant y \leqslant L_1$, and $0 \leqslant z \leqslant L_2$ stay constant as time passes, constantly increase, constantly decrease or fluctuate as time passes?

c) Such plane waves extend infinitely in all directions. What would the energy contained in a such a wave be?