Phys 473
Fall 2015

Modern Optics: Homework 4

Due: 28 August 2015

1 Bennett, *Principles of Physical Optics*, 2.4, page 32.

2 Induced electric field

Consider a cylindrically symmetric magnetic field that depends on time in a region in which there are no source charges or currents. Thus, using the usual cylindrical coordinates,

$$\mathbf{B} = B_z(s,t)\,\hat{\mathbf{z}}.$$

This will induce an electric field. The resulting electric and magnetic fields must satisfy Maxwell's equations.

a) Show that the magnetic field satisfies

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0.$$

b) Use

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

to determine a general expression for the induced electric field. Verify that the result satisfies

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

and also

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}.$$

c) The remaining Maxwell equation that these fields must satisfy is

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Substitute the electric field that you obtained in a previous part back into this to show that the magnetic field must satisfy

$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial B_z}{\partial s}\right) = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}.$$

d) Suppose that the magnetic field is uniform, i.e. it does not depend on position. Use the result for the previous part to show that, at most, the field can vary linearly with respect to time.

3 Bennett, *Principles of Physical Optics*, 2.5, page 38.

4 Bennett, *Principles of Physical Optics*, 2.6, page 38.

5 Bennett, *Principles of Physical Optics*, 2.21, page 51. *Hint: Start with a plane wave solution and substitute into one of Maxwell's equations.*