Modern Optics: Homework 3

Due: 24 August 2015

1 Bennett, *Principles of Physical Optics*, 1.20, page 22.

2 Three dimensional plane waves

a) The wave vector for a three dimensional wave is $\mathbf{k} = 40\,\mathrm{m}^{-1}\,\hat{\mathbf{i}} - 30\,\mathrm{m}^{-1}\,\hat{\mathbf{j}}$ and the angular frequency of the wave is 200 Hz. Describe the direction in which the wave propagates and the speed with which it propagates. Sketch, as accurately as possible in the xy plane, the locations of the crests (maxima) of these waves at $t = 0$ assuming that the phase is $\phi = 0$. Determine the distance between adjacent crests.

b) A plane wave has crests that are parallel to the z axis and also the line $y = \frac{1}{2}\,x$. The crests are spaced 0.10 m apart and the wave travels with speed 4000 m/s. Determine the wave vector, $\mathbf{k}$ for this wave and the speed of the wave.

3 General solution to the three dimensional wave equation

a) Show that $f(\mathbf{k} \cdot \mathbf{r} - \omega t)$, where $f(u)$ is any function of a single variable, u, satisfies the three dimensional wave equation.

b) Suppose that $f(u) = Ae^{-u^2}$. Sketch, as accurately as possible in the xy plane, the location of the maximum displacement along this wave at $t = 0$ if $\mathbf{k} = k(\hat{\mathbf{i}} - \hat{\mathbf{j}})/\sqrt{2}$. In which direction does this propagate?

4 Spherical waves

Consider the spherical wave for which $k = 0.10\,\mathrm{m}^{-1}$ and which has angular frequency 50 Hz and phase $\varphi = 0$. Describe the locations of the crests of this wave at $t = 0$ (for $r > 0$). Counting the crests outward from the origin, determine the ratio of the amplitude of the second crest to the first crest. Repeat this for the third crest compared to the second.

5 Bennett, *Principles of Physical Optics*, 1.34, page 24.

6 General harmonic spherical waves

Suppose that

$$\tilde{\Psi}(r,t) = f(r)e^{i\omega t}$$

where f is any function. Use the result of problem 1.34 and to determine a general solution for $f(r)$ and show that it reduces to a harmonic spherical wave solution.