Modern Optics: Homework 1

Due: 19 August 2015

1 Solutions to the one dimensional wave equation

Check, by direct substitution, whether the following satisfy the classical wave equation.

a) $\Psi(x,t) = A\sqrt{x - vt}$ assuming that $x > vt$.

b) $\Psi(x,t) = A\left(x - \sqrt{vt}\right)$ where $v > 0$.

2 Bennett, *Principles of Physical Optics*, 1.3, page 8. After solving this problem, consider the special case where $a = b = 1$ and $A = 1$. Plot this (over the range $-10 \leqslant x \leqslant 10$) for $t = 0$ and $t = 2$. Does the form of the graph remain the same? How does the graph change over the period from $t = 0$ to $t = 2$?

3 Bennett, *Principles of Physical Optics*, 1.4, page 8.

4 One dimensional wave equation: separation of variables

Assume that the solution to the one dimensional wave equation can be written as

$$\Psi(x,t) = f(x)T(t)$$

where $f(x)$ is any function of position and $T(t)$ is any function of t.

a) Substitute this into the wave equation and manipulate this to give and expression of the form:

derivatives and functions only depending on $x =$ derivatives and functions only depending on t.

b) The previous can only be true if either side is independent of both x and t, i.e. is a constant. Set the constant equal to $-k^2$ where k is real, i.e.

$$\text{derivatives and functions only depending on } x = -k^2$$

and solve the resulting differential equation for $f(x)$.

c) Substitute your solution into $\Psi(x,t) = f(x)T(t)$ and then resubstitute this into the wave equation. Solve the resulting equation for $T(t)$.

d) Combine your results to obtain the general separable solution $\Psi(x,t) = f(x)T(t)$.