Phys 473
Fall 2015

Modern Optics: Class Exam II

9 November 2015

Name: ____________________________ **Total:** **/50**

Instructions

- There are 5 questions on 6 pages.
- Show your reasoning and calculations and always justify your answers.

Physical constants and useful formulae

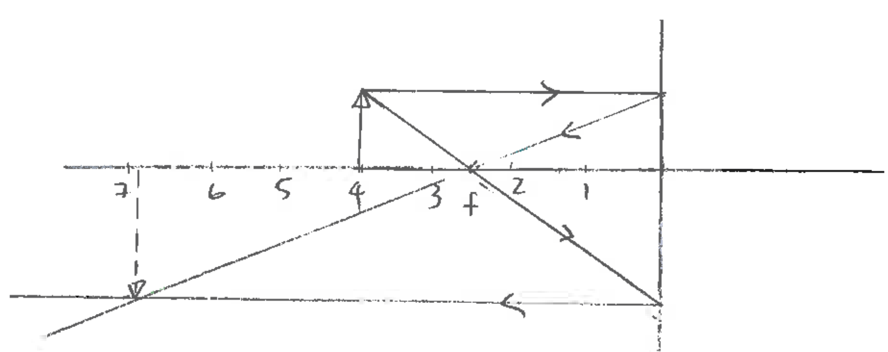
$$c = 3.0 \times 10^8\,\text{m/s} \qquad \epsilon_0 = 8.85 \times 10^{-12}\,\text{C}^2/\text{Nm}^2 \qquad \mu_0 = 4\pi \times 10^{-12}\,\text{Tm/A}$$

Question 1

A spherical mirror has a concave surface, whose radius of curvature is 5.0 cm. An object is placed at a distance of 4.0 cm from the surface of the mirror. Determine the location of the image *using equations* and also *using a ray tracing diagram.*

+1 $\left[\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f} \right.$ where $\left. f = R/2 = 2.5\text{cm} \right]$ +1

$\Rightarrow \frac{1}{4.0} + \frac{1}{S_i} = \frac{1}{2.5} \Rightarrow \frac{1}{S_i} = \frac{1}{2.5} - \frac{1}{4.0} \Rightarrow S_i = \frac{10}{1.5} = 6.67$] +2

+4 

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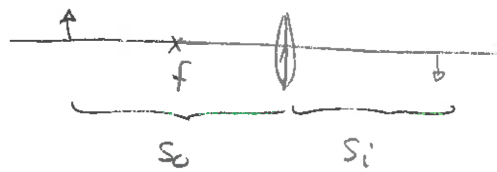
Question 2

An object is located beyond the focal length of a converging lens which produces an image on a screen. The location of the object and the screen are fixed and the distance between them is D. The position of the lens can be adjusted. Let s_o be the distance from the lens to the object and f be the focal length of the lens. Determine a general expression for s_o in terms of D and f that will result in a clear image on the screen.

+1 $D = S_o + S_i$



$+\cancel{3}4$

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{S_i} = \frac{1}{f} - \frac{1}{S_o} = \frac{S_o - f}{f S_o} \Rightarrow S_i = \frac{f S_o}{S_o - f}$$

+5

$$D = S_o + \frac{f S_o}{(S_o - f)} = S_o\left(\frac{S_o - f + f}{S_o - f}\right)$$

$$= \frac{S_o^{2}}{S_o - f}$$

$\Rightarrow$

$$D(S_o - f) = S_o^{2} \Rightarrow S_o^{2} - D S_o + Df = 0$$

$$\Rightarrow S_o = \frac{D \pm \sqrt{D^2 - 4Df}}{2} = \frac{D}{2}\left(1 \pm \sqrt{1 - 4f/D}\right)$$

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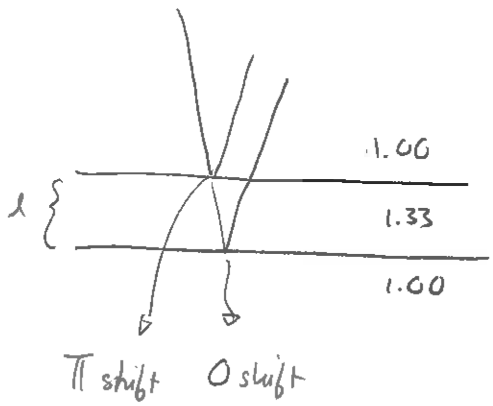
Question 3

A thin film of soapy water ($n = 1.33$) is held vertically in air ($n = 1.00$). Light of wavelength 590 nm is incident perpendicularly to the surface of the soap film. Determine the minimum thickness of the film which results is strongly reflected light.



Constructive interference

$$\Delta\phi = 2n\pi \qquad n = 0, \pm 1, \ldots$$

between two reflected waves

$$\Delta\phi = \Delta\phi_{path} + \Delta\phi_{refl.}$$

$$= \frac{2l}{\lambda_n} 2\pi + \Delta\phi_{refl.}$$

$$\Delta\phi_{refl} = -\pi$$

$$\Delta\phi = \frac{2l}{\lambda_n} 2\pi - \pi \qquad \lambda_n = \lambda/n$$

$$\Rightarrow \quad 2n\cancel{\pi} = \cancel{\pi}\left(\frac{4ln}{\lambda} - 1\right)$$

Smallest occurs when $\frac{4ln}{\lambda} = 1 \Rightarrow \quad l = \frac{\lambda}{4n} = \frac{590\,nm}{4 \times 1.33} = 111\,nm$

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Question 4

Two plane wave sources have the same linear polarization and oscillate in phase with the same wavenumber, k. The waves propagate along the x axis. One of the sources is located at x_1 and the other at $x_2 > x_1$.

a) Show that the time averaged irradiance at a detector located at $x > x_2$ is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(k\Delta x)$$

where $\Delta x = x_2 - x_1$ and I_j is the intensity of source j in the absence of the other source.

$$E = E_1 + E_2$$

$$E_1 = E_{01} e^{i(k(x-x_1)-\omega t)} \qquad E_2 = E_{02} e^{i(k(x-x_2)-\omega t)}$$

↑ real

$$I = \frac{1}{2}\epsilon v \; E \cdot E^* = \frac{1}{2}\epsilon v \left(E_1 + E_2\right)\left(E_1^* + E_2^*\right)$$

$$= \underbrace{\frac{1}{2}\epsilon v E_1 \cdot E_1^*}_{I_1} + \underbrace{\frac{1}{2}\epsilon v \, E_2 \cdot E_2^*}_{I_2} + \frac{1}{2}\epsilon v \left(E_1 E_2^* + E_1^* E_2\right)$$

Now $\quad E_1 E_2^* = E_{01}E_{02} \, e^{i(k(x-x_1)\cancel{-\omega t} - k(x-x_2) \cancel{+\omega t})}$

$$= E_{01}E_{02} \, e^{ik(x_2-x_1)} = E_{01}E_{02} \, e^{ik\Delta x}$$

$$E_2 E_1^* = E_{01}E_{02} \, e^{-ik\Delta x}.$$

$$\Rightarrow I = I_1 + I_2 + \epsilon v E_{01} E_{02} \cos(k\Delta x)$$

$$\frac{1}{2}\epsilon v E_{01}^2 = I_1$$

$$\Rightarrow E_{01} = \sqrt{2I_1/\epsilon v}$$

$$= I_1 + I_2 + 2\sqrt{I I_2}\cos(k\Delta x)$$

Question 4 continued ...

b) Suppose that the distance between the sources can be varied. Determine the visibility of the resulting interference pattern if $I_1 = 4I_2$.

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$I_{max} = \left(\sqrt{I_1} + 2\sqrt{I_1}\right)^2 = 9 I_1$$

$$I_{min} = \left(\sqrt{I_1} - 2\sqrt{I_1}\right)^2 = I_1$$

$$V = \frac{8}{10}$$

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Question 5

A wave pulse is represented in complex form via

$$\psi(t) = Ae^{-|t|/\tau}e^{i\omega_0 t}$$

where $\tau > 0$ and ω_0 are constants. Determine the Fourier transform of $\psi(t)$.

$$\tilde{\Psi}(k) = \int_{-\infty}^{\infty} \Psi(t) e^{-i\omega t} dt = A \int_{-\infty}^{\infty} e^{-|t|/\tau} e^{i(\omega-\omega_0)t} dt$$

$$= A\int_0^\infty e^{-t/\tau} e^{i(\omega-\omega_0)t} dt + A\int_{-\infty}^{0} e^{t/\tau} e^{i(\omega-\omega_0)t} dt$$

$$= A\int_0^\infty e^{t[i(\omega-\omega_0)-1/\tau]} dt + A\int_{-\infty}^{0} e^{t[i(\omega-\omega_0)+\frac{1}{\tau}]} dt.$$

$$= A \left. \frac{e^{t[\dots]}}{i(\omega-\omega_0)-1/\tau} \right|_0^\infty + A \left. \frac{e^{t[\dots]}}{i(\omega-\omega_0)+1/\tau} \right|_\infty^0$$

$$= \frac{-A}{i(\omega-\omega_0)-1/\tau} + \frac{A}{i(\omega-\omega_0)+\frac{1}{\tau}}$$

$$= A\left\{ \frac{-\cancel{i(\omega-\omega_0)}-\frac{1}{\tau}+\cancel{i(\omega-\omega_0)}-\frac{1}{\tau}}{\left(i(\omega-\omega_0)+\frac{1}{\tau}\right)\left(i(\omega_0-\omega_0)-\frac{1}{\tau}\right)} \right\}$$

$$= -\frac{A2}{\tau} \frac{1}{-\left(\frac{1}{\tau}\right)^2-(\omega-\omega_0)^2}$$

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$$\tilde{\Psi}(k) = \frac{2A}{\tau\left[(\omega-\omega_0)^2+\frac{1}{\tau^2}\right]}$$