Electromagnetic Theory: Homework 22
Due: 18 November 2014

1 Continuity equation: uniform density
An infinitely long cylinder of radius $R$ carries a uniform charge density $\rho$. The cylinder is dragged along the direction of its axis with speed $v$. Determine the current density and show that it satisfies the continuity equation.

2 Continuity equation: non-uniform density
An infinitely long cylinder of radius $R$ carries fixed charges. The charge density at $t = 0$, given in cylindrical coordinates, is
\[ \rho(r) = e^{-z^2/a^2} \]
where $a$ is a constant with units of length. The cylinder is dragged with speed $v$ long its axis.

   a) Sketch a plot of the charge density as a function of $z$ at $t = 0$. At which value of $z$ is the density greatest?

   b) Sketch a plot of the charge density as a function of $z$ at $t = 1$. At which value of $z$ is the density greatest?

   c) Show that the density at any later time is
\[ \rho(r, t) = e^{-(z-vt)^2/a^2}. \]

   d) Determine an expression for the current density at any time and show that it satisfies the continuity equation.

3 Current in Ohmic conductors
Show that for any Ohmic conductor, the total current through any surface is
\[ I = \sigma \int_{\text{Surface}} \mathbf{E} \cdot d\mathbf{a}. \]
where $\sigma$ is the conductivity.

4 Griffiths, Introduction to Electrodynamics, 7.1, page 301. First you must prove that between the spheres the potential is
\[ V(r) = \frac{V(b) - V(a)}{b - a} \left[ 1 - \frac{a}{r} \right] + V(a). \]
Prove this by showing that $V$ satisfies the Poisson equation in the region between the spheres and that it gives the correct results on the boundaries of this region. Then use this to find the electric field between the spheres and use the result from the previous exercise to determine the total current that flows.