1 Electric field and potential

A solid sphere of radius $R$ carries charge whose density is

$$\rho = \alpha r'$$

where $\alpha$ is constant.

a) Determine an expression for the total charge, $Q$, in the sphere and use this to express $\alpha$ in terms of $Q$.

b) Use Gauss’s Law to determine the electric field anywhere inside or outside the sphere. Start with the field inside the sphere. Your solution must contain in the following order:

i) A diagram of the charge distribution.

ii) A simplification of the electric field using symmetry arguments.

iii) An illustration and explanation of what Gaussian surface (the surface that appears in the surface integral) you are using. If this is the same as the surface of the charged sphere then all that you will calculate is the field on the surface of the sphere. That would be incomplete and incorrect. The illustration must indicate the enclosed charge.

iv) Evaluation of the surface integral.

v) Evaluation of the enclosed charge.

You will have to repeat all but the first two parts when you determine the field outside the sphere.

c) Use the field that you calculated to determine the electrostatic potential at any location, setting $V = 0$ at any location infinitely far from the sphere.

d) Use the potential to compute the electric field. Check that your results are correct.

e) Check that Poisson’s equation is correct for this distribution.

2 Work and kinetic energy in electrostatics

A disk of radius $R$ has uniform positive surface charge density $\sigma$. The disk lies in the $xy$ plane with its center at the origin. The potential at any point on the $z$ axis is

$$V = \frac{\sigma}{2\varepsilon_0} \left( \sqrt{R^2 + z^2} - z \right).$$

a) A positive test charge with charge, $Q$, and mass, $m$, is held at rest at height $h$ above the center of the disk. Determine an expression for the electrostatic potential energy of the point charge.
b) The charge is released. Determine an expression for its velocity when it is infinitely far from the disk.


5 Griffiths, *Introduction to Electrodynamics*, 2.32, page 94. This is a bonus problem, worth a total of 5 bonus points.