Electromagnetic Theory: Homework 6
Due: 9 September 2014

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10% (or for a single minor error 5%), point the errors out and you must submit a corrected assignment by 16 September 2014. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 23 September 2014. This will continue until you have solved every problem correctly.

1 Griffiths, *Introduction to Electrodynamics*, 1.34, page 36.

2 **Cylindrical unit vectors**

   Consider two points in the $z = 0$ plane. Denote the point $(1,1,0)$ by $P_1$ and the point $(1,-1,0)$ by $P_2$. Is $\hat{s}$ the same at $P_1$ as $P_2$? Is $\hat{\phi}$ the same at $P_1$ as $P_2$? Explain your answers.

3 **Stoke’s theorem: cylindrical coordinates**

   Consider the vector field, in cylindrical coordinates,
   \[ \mathbf{v} = z\hat{s} - s\hat{z}. \]
   a) Sketch the vector field in the $xz$ plane, i.e. at locations for which $\phi = 0$. Your sketch should include an $s$ axis, a $z$ axis and the vector field. Determine $\nabla \times \mathbf{v}$ and verify that the result is consistent with your sketch of the vector field.
   b) Verify that Stoke’s theorem applies to this field by using the path given in Cartesian coordinates as $(0,0,0) \rightarrow (0,0,1) \rightarrow (1,0,1) \rightarrow (1,0,0) \rightarrow (0,0,0)$.

4 **Stoke’s theorem: cylindrical coordinates**

   Consider the vector field, in cylindrical coordinates,
   \[ \mathbf{v} = s \cos \phi \hat{z} - z\hat{\phi}. \]
   Verify that Stoke’s theorem applies to this field by using the path given as follows:

   1. Follow an arc of radius 1 in the $xy$ plane starting from the positive $x$ axis and reaching the positive $y$ axis.
   2. Follow a vertical line in the $yz$ plane reaching $z = 2$.
   3. Follow an arc of radius 1 starting from the positive $y$ axis and returning to the positive $x$ axis by the shortest route.
   4. Follow a vertical line in the $xz$ plane reaching the initial starting point.
Note: this path describes a quarter of the curved surface of a cylinder whose axis is along the $z$ axis.

5 Griffiths, *Introduction to Electrodynamics*, 1.43a) and b), page 45.