Electromagnetic Theory: Homework 4
Due: 2 September 2014

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10% (or for a single minor error 5%), point the errors out and you must submit a corrected assignment by 9 September 2014. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 16 September 2014. This will continue until you have solved every problem correctly.

1 Laplacian
Calculate the Laplacian of

\[ f(x, y, z) = \sin (kx) \sin (ky)e^{-kz} \]

where \( k \) is a constant.

2 Charge density
Charge is distributed with density

\[ \rho = \frac{q}{a^3}x \]

where \( q \) is a constant with dimensions of charge.

a) Determine the total charge in the cube with sides of length \( a \) and corners at \((0, 0, 0), (0, 0, a), (0, a, 0), (0, a, a), (a, 0, 0), (a, 0, a), (a, a, 0), (a, a, a)\).

b) Determine \( \int \rho \, d\tau \) for the region which consists of half of a cylinder that is parallel to the \( z \) axis, ranges from \( z = 0 \) to \( z = a \), has radius \( a \) and whose base in the \( xy \) plane is as illustrated.
c) Explain without integrating what the total charge would be in a cylinder of the type
described above but whose base is a full circle in the $xy$ plane.


4 Line integrals in two dimensions

Let $\mathbf{v} = ay\mathbf{x} \mathbf{x} - x^2\mathbf{y}$. Let $a$ be a positive number. Three paths are indicated in the $z = 0$ plane below.

a) Determine the line integral of $\mathbf{v}$ along line 1.

b) Determine the line integral of $\mathbf{v}$ along line 2: in the $xy$ plane with straight line segments

$$(0, 0) \to (a, a) \to (a, -a).$$

c) Determine the line integral of $\mathbf{v}$ along line 3: in the $xy$ plane with straight line segments

$$(0, 0) \to (0, -a) \to (a, -a).$$