Electromagnetic Theory: Homework 2
Due: 26 August 2014

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 28 August 2014. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 2 September 2014. This will continue until you have solved every problem correctly.

1 Vector Triple Product

Let

\[ A = 4\hat{x} \]
\[ B = 2\hat{x} + 3\hat{y} \]
\[ C = 2\hat{x} - 3\hat{y} \]

Verify that these satisfy the rule

\[ A \times (B \times C) = B(A \cdot C) - C(A \cdot B) \]

by explicitly calculating all the terms on both sides.

2 Functions and Vector Fields

Consider a fluid that can flow through a three dimensional region of space. Any location in this region can be specified using coordinates \( x, y, z \). Assume that the fluid is continuous (does not consist of individual molecules) and that the temperature, pressure and velocity of the fluid can vary from one location to another. Describe which of the following are scalar functions (of the location \( x, y, z \)) and which are vector fields.

a) Volume of the entire fluid.
b) Mass of the entire fluid.
c) Temperature.
d) Pressure.
e) Velocity.

3 Gradient of a function

Let \( V(x, y) = x^2 + y^2 \).

a) Sketch several contours of \( V(x, y) \) in the \( xy \)-plane. Indicate which provide larger values and which provide smaller values.
b) Determine $\nabla V$ and sketch the resulting vector field on your contour plot.

\( V \) and sketch the resulting vector field on your contour plot.

\( \nabla V \) is perpendicular to the contours.

5 **Gradients of distances**

Consider a conventional coordinate system and suppose that \( \mathbf{r} \) is a position vector for the locations with coordinates \((x, y, z)\).

a) Using the general rule that the unit vector along \( \mathbf{A} \) is \( \hat{\mathbf{A}} = \mathbf{A}/A \), determine an expression for \( \hat{\mathbf{r}} \) in terms of \( x, y, z \) and the standard basis. Determine an expression for \( \mathbf{r} \) in terms of \( x, y \) and \( z \).

b) For any integer \( n \) show that \( \nabla (r^n) = nr^{n-1}\hat{\mathbf{r}} \). (Note that \( \hat{\mathbf{r}} \) is the unit vector along \( \mathbf{r} \).)

6 **Griffiths, Introduction to Electrodynamics, 1.13, page 15.**

This is similar to the previous problem, except that the position vector is replaced by a separation vector. In this case the differentiations within the gradient mean differentiation with respect to the unprimed coordinates. The primed coordinates can be treated as constants. This is an important result that we will use many times throughout this course.