Electromagnetic Theory: Class Exam II
15 November 2011

Name: ___________________________ Total: ________________ /50

Instructions
• There are 4 questions on 7 pages.
• Show your reasoning and calculations and always justify your answers.

Physical constants and useful formulae

Permittivity of free space $\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2$
Charge of an electron $e = -1.60 \times 10^{-19} \text{C}$

Integrals

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}$$
$$\int xe^{ax} \, dx = \frac{ax - 1}{a^2} e^{ax}$$
$$\int x^2 e^{ax} \, dx = \frac{a^2 x^2 - 2ax + 2}{a^3} e^{ax}$$
$$\int e^{ax} \, d\alpha = \frac{1}{a} e^{ax}$$

$$\int \sin (ax) \sin (bx) \, dx = \frac{\sin ((a - b)x)}{2(a - b)} - \frac{\sin ((a + b)x)}{2(a + b)} \quad \text{if} \ a \neq b$$

$$\int \cos (ax) \cos (bx) \, dx = \frac{\sin ((a - b)x)}{2(a - b)} + \frac{\sin ((a + b)x)}{2(a + b)} \quad \text{if} \ a \neq b$$

$$\int \sin (ax) \cos (ax) \, dx = \frac{1}{2a} \sin^2 (ax)$$

$$\int \sin^2 (ax) \, dx = \frac{x}{2} - \frac{\sin (2ax)}{4a}$$

$$\int \cos^2 (ax) \, dx = \frac{x}{2} + \frac{\sin (2ax)}{4a}$$

$$\int x \sin^2 (ax) \, dx = \frac{x^2}{4} - \frac{x \sin (2ax)}{4a} - \frac{\cos (2ax)}{8a^2}$$

$$\int x^2 \sin^2 (ax) \, dx = \frac{x^3}{6} - \frac{x^2}{4a} \sin (2ax) - \frac{x}{4a^2} \cos (2ax) + \frac{1}{8a^3} \sin (2ax)$$
Question 1

In the following question do either part a) or part b) for full credit. If you do both parts, each will be graded and you will be given the highest score that you obtained for one of the parts.

a) Two solid objects are made of a perfect conducting material and are neutral. Each contains a spherical cavity of radius $2.0 \times 10^{-2}$ m. A single point charge is placed at the center of each cavity. For object A this has charge $2.0 \times 10^{-6}$ C and for object B it has charge $4.0 \times 10^{-6}$ C. The objects, which are separated so as to have no effect on each other, are illustrated to scale in the diagram. For each object determine the potential on the surface of the cavity and the total charge on the outer surface of the object. Describe as precisely as possible how the potential on the outer surface of A compares to that on B.
b) A sphere of radius $R$ contains charge whose density is $\rho(r, \theta) = k \cos(\phi)$ where $r, \theta$ and $\phi$ are the standard spherical coordinates. Determine the electric dipole moment. Hint: consider the symmetry of the charge distribution.
An infinitely long cylindrical shell, of negligible thickness and radius $b$, surrounds an infinitely long solid cylindrical rod, of radius $a$. Their axes are both along the $z$-axis and a view down the length of this axis is illustrated. The volume current density in the rod is (in cylindrical coordinates) $J = \frac{3\alpha}{2\pi a^3} s\hat{z}$ where $\alpha$ is a constant with the dimensions of current. The current along the cylindrical shell, flows in the $-\hat{z}$ direction, is uniformly distributed across the surface and is such that the total current flowing down the shell is exactly opposite to that flowing through the cylinder.

a) Show that the surface current density in the cylindrical shell is $K = -\frac{\alpha}{2\pi b} \hat{z}$.
b) Determine the magnetic field at all locations.
Question 3

A magnetic vector potential in cylindrical coordinates is $A = ks \hat{\phi}$ where $k$ is a constant. Determine the magnetic field, $B$, produced by this and verify that $\nabla \cdot B = 0$. 
Question 4

A loop of radius $R$ carries charge of uniform linear density, $\lambda$, and lies in the $xy$ plane. The loop rotates about the axis through its center, as illustrated in the diagram, so that any point moves with speed $v$. Determine an expression for the magnetic dipole moment of the loop in terms of $R, \lambda, v$ and constants.