Electromagnetic Theory: Class Exam I
4 October 2011

Name: ___________________________ Total: /50

Instructions

• There are 3 questions on 6 pages.
• Show your reasoning and calculations and always justify your answers.

Physical constants and useful formulae

Permittivity of free space \( \epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2 \)
Charge of an electron \( e = -1.60 \times 10^{-19} \text{C} \)

Integrals

\[
\int e^{ax} \, dx = \frac{1}{a} e^{ax} \\
\int x e^{ax} \, dx = \frac{ax - 1}{a^2} e^{ax} \\
\int x^2 e^{ax} \, dx = \frac{a^2 x^2 - 2ax + 2}{a^3} e^{ax} \\
\int e^{ax} \, dx = \frac{1}{a} e^{ax}
\]
Question 1

A sphere of radius $R$ contains total charge that is distributed according to the charge density

$$\rho = kr$$

where $r$ is the distance from the center of the sphere and $k$ is a constant.

a) Suppose that the total charge contained within the entire sphere is $Q$. Determine an expression for $Q$ in terms of $k$ and $R$.

b) Determine expressions for the electric field at all points inside and outside the sphere.
Question 2

A particular electrostatic charge distribution gives an electric field, described in cylindrical coordinates, of

\[ \mathbf{E} = \frac{k}{s^2} \hat{s} \]

where \( k \) is a constant.

a) Determine the electrostatic potential at any point, taking the potential at infinity as zero.

b) A claim is made that this electric field can be produced by a charge distribution that is restricted to a region of space (i.e. for most locations the charge density is zero). Is this true?
Question 3

It is postulated that a static charge distribution gives rise to an electric field, described in spherical coordinates, of

\[ E = \alpha \frac{e^{-\lambda r}}{r} \hat{r} \]

where \( \alpha \) and \( \lambda \) are constants.

a) Show that this is a legitimate electric field and find the charge distribution which gives rise to this field.
b) Determine the work required to assemble (i.e. energy stored in) this charge distribution.