Lecture 43

Final: Mon 9 Dec 2pm

Covers: Waves/vibrations

HW 8-20

Lectures 24-43

Previous exams: 2012 Midterm 2 Q1 - Q5

Final Q1→,2,3,5

2011 Midterm 2 Q1 → Q5

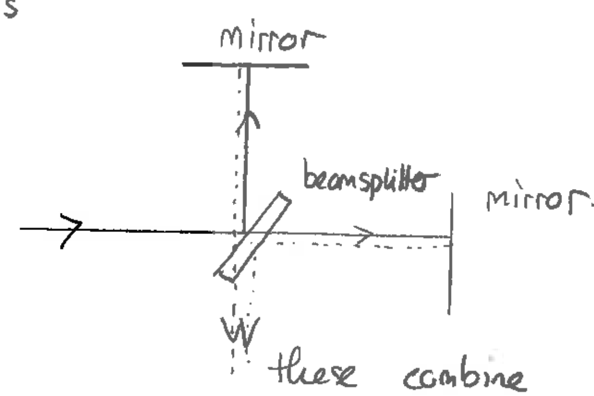
Final Q1,2,3,4

Bring Single letter sheet any eqns

Interferometry

One application of inteference particularly in optics involves splitting + recombining waves. In optics this is done with mirrors + beamsplitters. This is called interferometry.

The Michelson interferometer is as illustrated



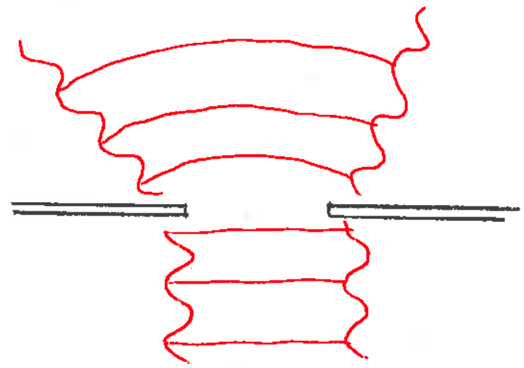
Quiz 1

Demo: LIGO site.

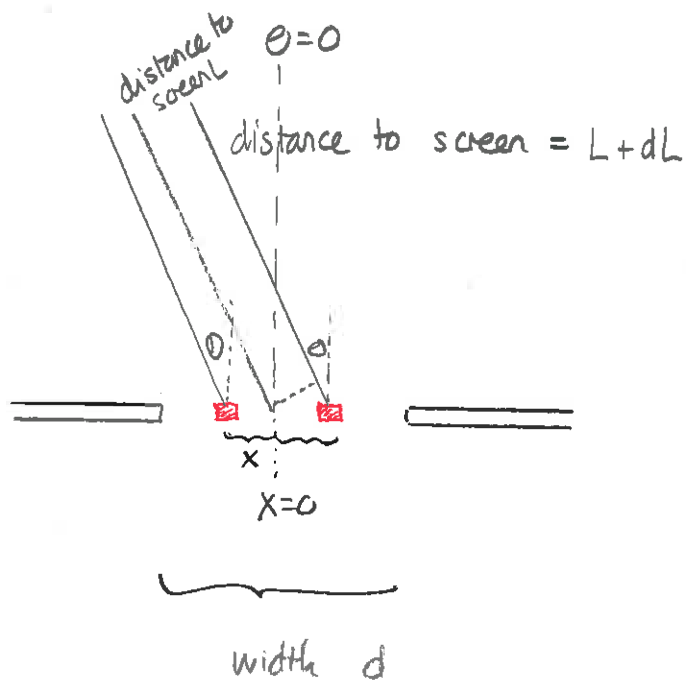
Diffraction:

One situation described by interference is that where waves are incident on an opening. The waves bend and produce a pattern beyond the opening. This is called diffraction.

Demo: Ocean waves



We describe this by noting that each point within the opening produces a wave. This infinite multitude of waves interfere and form a superposition. The set up is as illustrated with two waves from two portions of the opening illustrated.



Consider the contribution from the portion at x with width dx. This is a wave of the form:

$$A \cos\left[k(L+dL)-\omega t\right] dx$$

We need to add these

Exercise 1 Using the set up of the notes do:

a) express dL in terms of x and θ

b) express the wave in terms of a complex variable and integrate over the range of x to determine an expression for the superposition

c) determine an expression for the amplitude of the wave. Use this to determine an expression for the intensity of the wave.

Exercise 2

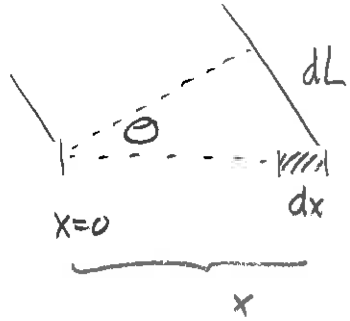
a) Determine conditions / locations at which a minimum of intensity will be attained.

b) Determine conditions for which there will be no minima from diffraction

Ans 1 :

a) $\frac{dL}{x} = \sin\theta$

$\Rightarrow dL = x\sin\theta$



b) $A\cos\left[kL + kx\sin\theta - \omega t\right]dx \quad \leadsto \quad z(t) = Ae^{i(kL + kx\sin\theta - \omega t)}dx$

$\Rightarrow$ total superposition $= Z(t) = A\int_{-d/2}^{d/2} e^{i(kL + kx\sin\theta - \omega t)}\,dx$

$$= Ae^{i(kL-\omega t)}\int_{-d/2}^{d/2} e^{(ik\sin\theta)x}\,dx$$

$$= Ae^{i(kL-\omega t)}\frac{1}{ik\sin\theta}\,e^{ik\sin\theta}\Big|_{-d/2}^{d/2}$$

$$= Ae^{i(kL-\omega t)}\frac{1}{ik\sin\theta}\left[e^{ik\sin\theta d/2} - e^{-ik\sin\theta d/2}\right]$$

$$= Ae^{i(kL-\omega t)}\frac{1}{ik\sin\theta}\,2i\sin\left[\frac{dk}{2}\sin\theta\right]$$

$$= 2Ae^{i(kL-\omega t)}\frac{\sin\left[\frac{kd}{2}\sin\theta\right]}{k\sin\theta}$$

$$= Ade^{i(kL-\omega t)}\frac{\sin\left[\frac{kd}{2}\sin\theta\right]}{\frac{kd}{2}\sin\theta}$$

Let $\beta = \frac{kd}{2}\sin\theta$. Then superposition $= Ade^{i(kL-\omega t)}\frac{\sin\beta}{\beta}$

Now amplitude is

$$|Z(t)| = Ad \left|\frac{\sin\beta}{\beta}\right|$$

Intensity = const × amplitude^2

$$= \text{const} \times A^2 d^2 \frac{\sin^2\beta}{\beta^2} \qquad \beta = \frac{kd}{2}\sin\theta$$

Note that as $\theta \to 0$, $\beta \to 0$. In this case

$$\frac{\sin\beta}{\beta} \to 1$$

and thus the intensity as $\theta \to 0$ is $I_0 = \text{const} \times A^2 d^2$. So

$$I = I_0 \frac{\sin^2\beta}{\beta^2} \qquad \text{where } \beta = \frac{kd}{2}\sin\theta$$

Ex 2: a) need $\beta = 0$ and so $\frac{kd}{2}\sin\theta = n\pi$ for $n = \pm1, \pm2, \pm3, \ldots$

$$\Rightarrow \frac{2\pi d}{\lambda 2}\sin\theta = n\pi \Rightarrow \sin\theta = n\lambda/d$$

b) If $\lambda/d > 1$ then the condition for a minimum is

$$\sin\theta = n\lambda/d > n > 1$$

and there is no θ for which this works.