Weds: HW
Read 7.1.1
7.2.1

Exercise: If $z = e^{i\alpha} + e^{i\beta}$ then determine $|z|$

Answer: $|z| = \sqrt{zz^*}$

Then
$$z^* = e^{-i\alpha} + e^{-i\beta}$$

and
$$zz^* = (e^{i\alpha} + e^{i\beta})(e^{-i\alpha} + e^{-i\beta}) = 2 + e^{i(\alpha-\beta)} + e^{i(\beta-\alpha)}$$
$$= 2 + e^{i(\alpha-\beta)} + e^{-i(\alpha-\beta)}$$
$$= 2 + \cos(\alpha-\beta) + i\sin(\alpha-\beta) + \cos(-(\alpha-\beta)) + \overbrace{i\sin(-(\alpha-\beta))}^{=-i\sin(\alpha-\beta)}$$
$$= 2 + 2\cos(\alpha-\beta)$$

But $\cos\theta = 2\cos^2\theta/2 - 1 \Rightarrow \cos(\alpha-\beta) = 2\cos^2\frac{\alpha-\beta}{2} - 1$

So
$$zz^* = 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

and
$$|z| = \sqrt{zz^*} = 2\left|\cos\frac{\alpha-\beta}{2}\right|$$

III. Interference

There are many situations in which more than one source produces waves in the same medium at the same time. Such waves typically overlap and interfere. Some of the most striking wave phenomena arise from such interference.

1 Interference from two sources

Suppose that two sources produce waves that travel to the right as illustrated in Fig. III.1.1. At all points beyond source 2 there are two overlapping waves on the medium.

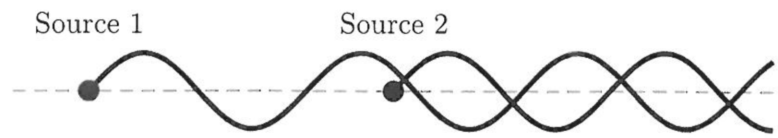

Figure III.1.1: Overlapping waves produced by two sources

The displacement of the wave produced by source 1 will be denoted $y_1(x,t)$ and that of source 2, $y_2(x,t)$. The displacement of the medium is then the *superposition* of that of the two waves:

$$\boxed{y(x,t) = y_1(x,t) + y_2(x,t).} \tag{III.1.1}$$

We shall initially consider situations where the two waves have the same amplitudes. We can immediately illustrate some extreme examples of such superpositions. *Constructive interference* occurs when the two waves overlap perfectly and this results in a wave of larger amplitude than either of the two waves. This is illustrated in Fig. III.1.2.

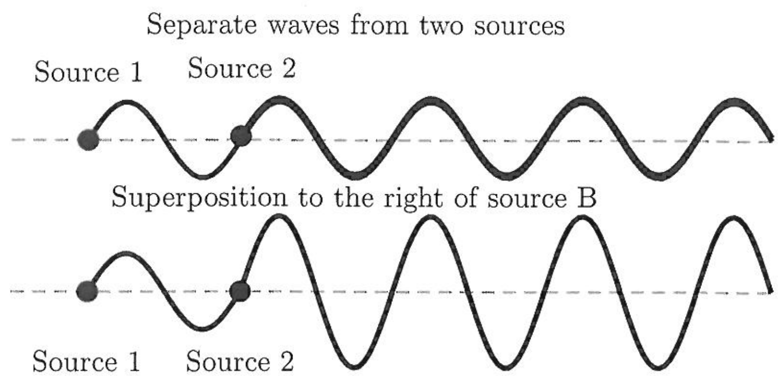

Figure III.1.2: Constructive interference of two waves.

Alternatively when the two waves are perfectly mismatched, Eq. (III.1.1) shows that cancellation occurs. This is called *destructive interference* and is illustrated in Fig. III.1.3. Between these extremes there is a continuous range of interference possibilities.

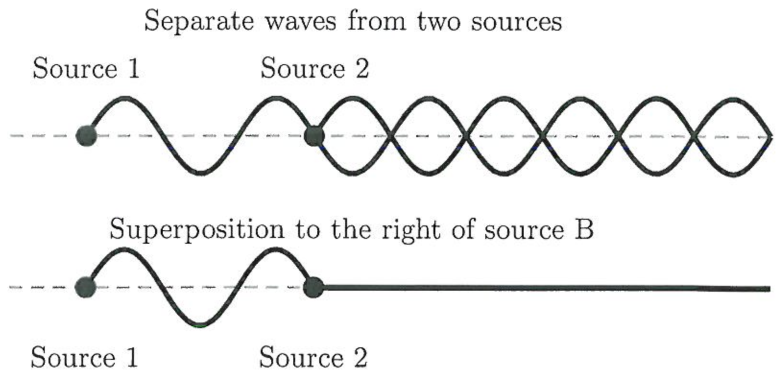

Figure III.1.3: Destructive interference of two waves.

The type of interference that is produced by two sources depends on the separation between the two sources as well as the relative phases of the two sources. Initially we shall consider situations in which the two sources oscillate in phase. This means that both sources oscillate at the same frequency and whenever source 1 attains a maximum vertical displacement so does source 2.

→ Slides 1-4.

Here constructive interference clearly results when the separation between the two sources, Δx, is an integral number of wavelengths. Similarly destructive interference results when the separation is a half integral number of wavelengths.

> Constructive interference occurs when
>
> $$\Delta x = n\lambda$$
>
> and destructive interference occurs when
>
> $$\Delta x = \left(n + \frac{1}{2}\right)\lambda \qquad \text{(III.1.2)}$$
>
> where λ is the wavelength of the waves.

→ Quiz 1 mixed → 100%
Quiz 2 → reward.

In general, sources will be positioned relative to each other so that neither perfect constructive nor destructive interference occurs. The more general case can be analyzed by obtaining expressions for the displacement produced by each wave which refer to the source location. Figure III.1.4 illustrates a source located at x_0.

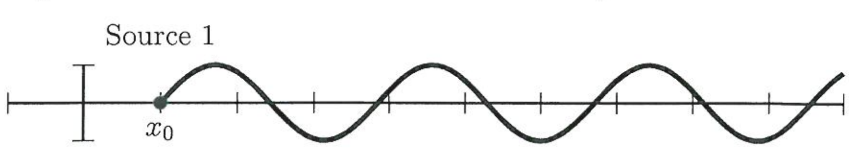

Figure III.1.4: General source.

Suppose that the source oscillates with frequency ω and amplitude A and that the wavespeed is v. The vertical displacement of the medium at the source location is

$$y_{\text{source}}(t) = A\cos(\omega t - \phi)$$

This disturbance takes time $(x - x_0)/v$ to reach any location, at position x, to the right of the source. Thus the displacement at x at time t is equal to that at the source at time $t - (x - x_0)/v$, or

$$\begin{aligned} y(x,t) &= A\cos\left[\omega(t - (x - x_0)/v) - \phi\right] \\ &= A\cos\left[\omega t - k(x - x_0) - \phi\right] \end{aligned}$$

where we have used $\omega = kv$. Rearranging gives:

> If a source at x_0 oscillates according to
>
> $$y_{\text{source}}(t) = A\cos(\omega t - \phi)$$
>
> then the displacement of the medium at any location $x > x_0$ at time t is
>
> $$y(x,t) = A\cos\left[k(x - x_0) - \omega t + \phi\right]$$
>
> where $k = \omega/v$ is the wavenumber of the waves with v being the wavespeed.

(III.1.3)