Phys 230 2013

Lecture 36

Mon: HW 16 due

Mon: Read 5.5, 5.6

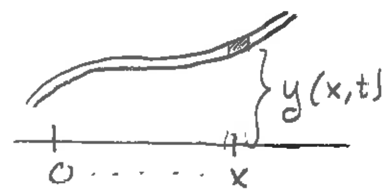
Sinusoidal Traveling Waves

General wave disturbances are represented by a displacement function of two variables

a) position, x, along string/medium

b) time, t

The displacement of the string at location x at time t is represented by: a function of two variables, $y(x,t)$.



A particular class of traveling wave consists of sinusoidal waves such as

$$\boxed{\begin{array}{ll} y(x,t) = A\sin[k(x-vt)+\phi] & \text{moves right} \\ y(x,t) = A\sin[k(x+vt)+\phi] & \text{moves left} \end{array}}$$

We were able to interpret the constants as follows

$\boxed{v = \text{wavespeed} = \text{speed with which disturbance propagates}}$

$\boxed{A = \text{amplitude} = \text{maximum displacement from equilibrium}}$

The horizontal extent of the wave pattern is described by

$\boxed{\lambda = \text{wavelength} = \text{distance between successive peaks in displacement.}}$

We can show that the wavenumber satisfies

$$\boxed{k = 2\pi/\lambda}$$

When observing a single point, we can see that this oscillates. For example, with the left traveling wave, at $x=0$, the displacement is

$$y(0,t) = A\sin(kvt+\phi)$$

Then we obtain a general rule.

> Each point on the medium oscillates with the same angular frequency
>
> $$\omega = kv.$$

In terms of frequency and wavelength one can show

$$\boxed{v = \lambda f.}$$

General Traveling Waves

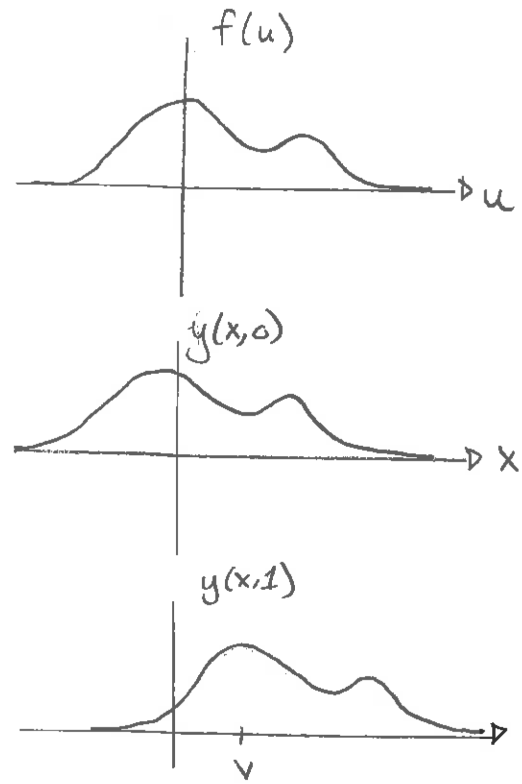
The wave forms that we have considered so far are two examples of functions of the form

$$y(x,t) = f(x-vt)$$

or

$$y(x,t) = g(x+vt)$$

where $f(u)$ and $g(u)$ are any functions of a single variable. We can see that these represent disturbances which propagate with unchanged shape. To see this note that at time $t+\Delta t$



$$y(x, t+\Delta t) = f(x-v(t+\Delta t))$$
$$= f(x - v\Delta t - vt)$$
$$= y(x-v\Delta t, t)$$

So to get the displacement at x at time $t+\Delta t$, we need only find the displacement at earlier time t but at the location $x-v\Delta t$. So in time Δt the entire pattern shifts right by distance $v\Delta t$. Thus

> For any $f(u)$, $g(u)$
>
> $$y(x,t) = f(x-vt)$$
>
> represents a disturbance traveling right and
>
> $$y(x,t) = g(x+vt)$$
>
> a disturbance traveling left

Either of these can be show to satisfy the partial differential equation:

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$

This is called the classical wave equation.

Waves on a String

Can we derive the classical wave equation for a physical system by starting with a fundamental principle such as Newton's 2nd Law?

In many cases, yes! For example consider a string with linear mass density, μ. (i.e. mass per unit length $= \mu$) and under tension T.

We can apply Newton's 2nd Law to individual segments. The result is that the displacement away from equilibrium satisfies:

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu} \frac{\partial^2 y}{\partial x^2}$$

and this is the classical wave equation with wave speed

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

Q1 a) $y(x,0) = A \sin kx$

b) $kx = \pi/2, \frac{5\pi}{2}, \frac{9\pi}{2} \Rightarrow x = \frac{\pi}{2k}, \frac{5\pi}{2k}, \frac{9\pi}{2k}$

adjacent maxima

The distance between adjacent maxima is: $\lambda = \frac{5\pi}{2k} - \frac{\pi}{2k}$

$= \frac{4\pi}{2k} = \frac{2\pi}{k}$

$\Rightarrow \lambda = \frac{2\pi}{k}$ or $k = \frac{2\pi}{\lambda}$

c) $y(0,t) = A \sin(kvt) \leftarrow$ oscillation with frequency $\omega = kv$

d) $2\pi f = \omega \Rightarrow 2\pi f = \frac{2\pi}{\lambda} v \Rightarrow \lambda f = v$

Q2

$$\frac{\partial y}{\partial t} = \omega A \sin(kx-\omega t) \qquad \frac{\partial y}{\partial x} = k \quad A\cos(kx-\omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos(kx-\omega t) \qquad \frac{\partial^2 y}{\partial x^2} = -k^2 \quad A\sin(kx-\omega t).$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{-\omega^2}{-k^2}\left(-k^2 A\sin(kx-\omega t)\right)$$

$$= \frac{\omega^2}{k^2}\frac{\partial^2 y}{\partial x^2}$$

Wave equation with $v^2 = \omega^2/k^2 \Rightarrow \omega = kv$

Q3 a)

$$\frac{\partial y}{\partial t} = An(x-vt)^{n-1}(-v)$$

$$\frac{\partial^2 y}{\partial t^2} = -Anv\,(-v)(n-1)(x-vt)^{n-2}$$

$$= (-v)^2 An(n-1)(x-vt)^{n-2} = v^2 An(n-1)(x-vt)^{n-2}$$

$$\frac{\partial y}{\partial x} = An(x-vt)^{n-1}$$

$$\frac{\partial^2 y}{\partial x^2} = An(n-1)(x-vt)^{n-2}$$

Clearly $\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$