Phys 230 2012

Lecture 35

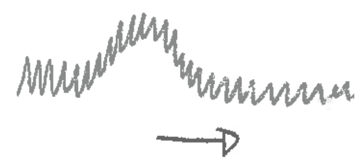
Friday: Read

Next HW: Monday

Waves:

A wave is a type of co-ordinated disturbance of a particular supporting medium. A typical example is a 'pulse' that travels down a slinky

Demo: Produce pulse along slinky



Demo: PhET Waves on a String

* no end * produce manual pulse

Here the slinky forms the "medium" that hosts the wave.

As the wave disturbance passes, the medium is disturbed from its equilibrium position. We can observe two general features:

1) as time passes the pattern propagates along the medium.
2) as time passes an individual "segment" of the medium moves.

Demo: PhET W.O.S focus on one bead

In these situations the bead / segment of the medium moves in a perpendicular direction to the pulse. This is called a transverse wave. Although the actual physical objects here are the beads / slinky coils we will eventually come to regard the wave disturbance itself as a physical object.

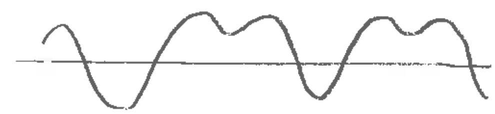
Quiz 1

Mathematical description of waves

Consider waves on a stretched string. There are two distinct aspects to the wave:

1) spatial: at any instant different locations on the string can have different displacements. A "snapshot" would reveal this

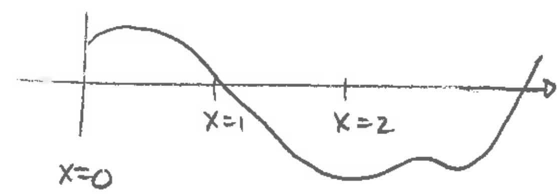
profile at one time.



2) temporal: as time passes the displacement of any point on the string fluctuates

We describe these by a function of two variables:

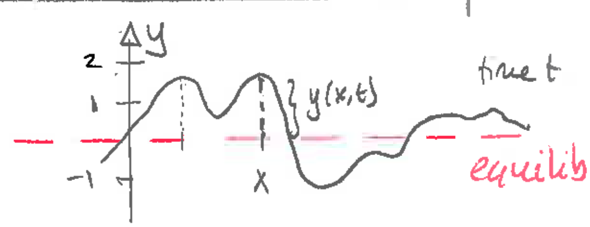
1) location along string: x
2) time : t



Then the wave is completely described by

The displacement away from equilibrium at location x and time t is denoted by $y(x,t)$

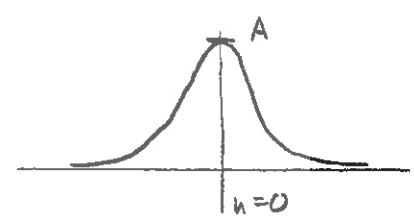
This entire function of two variables captures the full behavior of the wave.



Example: Gaussian Pulse

One example of a wave is that corresponding to the pulse generated by a Gaussian function

$$f(u) = Ae^{-u^2/a^2}$$



This has a peak at $u=0$, where $f(u)=A$

We can generate a wave from this by replacing

$$u \to x-vt$$

where v is a positive constant.

This gives a candidate example of a pulse-like wave:

$$y(x,t) = A e^{-(x-vt)^2/a^2} \qquad \text{"Gaussian pulse"}$$

There are many other possible pulses. We can illustrate this with snapshots at different times

Slide 1

We see that:

1) the shape of the pulse stays the same as time passes
2) the peak of the pulse is attained when $x - vt = 0$. Thus at any time t, the location of the peak is given by $x = vt$. As t increases x increases proportionally, with $\Delta x / \Delta t = v$. So v represents the speed of the wave.

Thus

> If $y(x,t) = A e^{-(x-vt)^2/a^2}$ then the disturbance propagates right with speed v. The peak amplitude is A

Quiz 2

We can see

> If $y(x,t) = A e^{-(x+vt)^2/a^2}$ then the disturbance propagates left with speed v.

We can extract information about the transverse speed v_y with which any point moves:

$$v_y = \frac{\partial y}{\partial t}$$

Quiz 3

$$\frac{\partial y}{\partial t} = -2(x-vt)(-v)\, A e^{-(x-vt)^2/a^2}$$

$$\left.\frac{\partial y}{\partial t}\right|_{t=0} = 2vx\, e^{-x^2/a^2}$$

Traveling sinusoidal waves

Mathematically a more convenient wave is one in which one end of the string is made to oscillate repeatedly. This produces a repetitive pattern

Demo: PhET w.o.s oscillate with open end  pattern.

Waves of this type must be described by a sinusoidal function. A snapshot of a sinusoidal function at $t=0$ would reveal:

$$y(x,0) = A\sin(kx+\phi)$$

for some constants A, k, ϕ. What could this give at later times. If the pattern is to shift a possible modification would be $x \to x \pm vt$. This gives:

Left moving sinusoidal traveling wave	$y(x,t) = A\sin[k(x-vt)+\phi]$
Right moving - - - - - -	$y(x,t) = A\sin[k(x+vt)+\phi]$

The same reasoning as before leads to:

> The constant v = speed with which pattern propagates
> (wave speed, phase velocity)

Then ϕ establish the horizontal offset and is called the phase of the wave. The other constants are A and k. The first is readily seen to quantify the transverse extent of this wave. More precisely

A = maximum displacement attained by any point

Demo PhET w.o.s, show amplitude

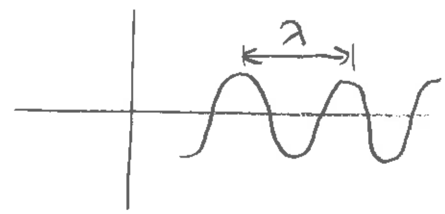
The remaining quantity captures details about the horizontal extent of the pattern.

Wavenumber, wavelength, frequency

The same pattern is repeated over + over. The wavelength quantifies the extent of a basic unit in the pattern. Precisely:

> The wavelength, λ, is the distance between successive peaks.

This is measured in meters.



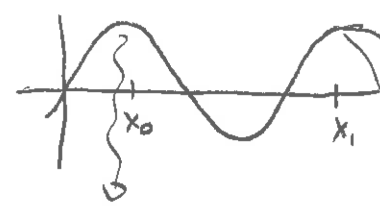
Slide 2

This must be related to k. We can show

$$\boxed{k = \frac{2\pi}{\lambda}}$$

Proof: Suppose one crest occurs at x_0 and the next at x_1. Then

$$\underbrace{k(x_0 - vt) + \phi}_{\text{original argument}} + \underbrace{2\pi}_{\substack{\text{step of } 2\pi \\ \text{again returns crest}}} = k(x_1 - vt) + \phi$$



here
$k(x_0 - vt) + \phi = \pi/2$

$k(x_1 - vt) + \phi = \pi/2 + 2\pi$
$= k(x_0 - vt) + \phi + 2\pi$

$$\Rightarrow k(x_0 - x_1) + 2\pi = 0$$

$$\Rightarrow x_1 - x_0 = 2\pi/k \Rightarrow \lambda = 2\pi/k$$

$$\Rightarrow k = 2\pi/\lambda.$$

The quantity k is called the <u>wavenumber</u> and it determines the spatial frequency of the wave.

Now consider a single point on the string. For convenience $x=0$. Then

$$y(0,t) = A \sin\left[k(0-vt)+\phi\right]$$

$$= -A \sin\left[kvt - \phi\right]$$

This point oscillates and the same is true of all other points. We can do this for a left moving disturbance

Quiz 4

We see that the angular frequency is

$\boxed{\omega = kv}$ "dispersion relation"

Thus we can rewrite the sinusoidal wave expressions:

left moving	$y(x,t) = A\sin(kx - \omega t + \phi)$
right "	$y(x,t) = A\sin(kx + \omega t + \phi)$