Phys 230 2013

Lecture 24

Mon HW due

read: King Ch 1.2.4, 1.2.5, 1.3.1 → 1.3.3

Exam: Average 70%

Vibrations + Oscillations

Consider a mass that is suspended from a spring. If the mass is pulled down and released, then it will subsequently repeatedly bounce up and down. Motion of this type in which a basic pattern is repeated over and over is called an oscillation. Examples of oscillations include:

1) mass / spring systems
2) pendulum
3) currents + voltages in circuits
4) vibrations in solids / crystal lattices
5) wave motion
6) quantum theory descriptions of light.

We shall consider examples of oscillations taken from classical physics. We aim to use Newtonian physics to describe these but will develop a language suitable for describing oscillatory phenomena.

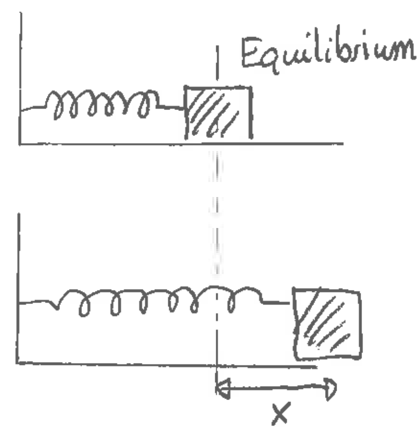
Mass and Spring

A classical example of an oscillating system is a mass attached to a spring. Suppose that the mass is supported by a horizontal frictionless surface. We can assess the situation using Newtonian mechanics. Schematically

Forces on system $\rightsquigarrow$ Equation of motion $\rightsquigarrow$ Solve equation of motion

We describe the system configuration via one variable

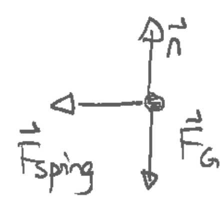
$$\boxed{x = \text{displacement from equilibrium}}$$



Ultimately we want to know how x depends on time.

Newtonian mechanics starts with

$$\vec{F}_{net} = m\vec{a}$$



For purely horizontal motion $\vec{n} = -\vec{F}_G$ and $\vec{F}_{net} = \vec{F}_{spring}$. So

$$\vec{F}_{spring} = m\vec{a}$$

We only need to consider the horizontal component of motion. So

$$F_{spring\,x} = ma_x$$

But

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2}$$

Also by Hooke's Law

$$F_{spring\,x} = -kx$$

where k is the spring constant. Substitution gives:

$$-kx = m\frac{d^2x}{dt^2}$$

which results in the equation of motion for the system

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x} \qquad -(1)$$

The equation of motion is a differential equation with:

1) two constants k, m that do not depend on t but are determined by the actual physical system.

2) A function $x = x(t)$ which depends on time

The equation of motion does not immediately yield the position as a function of time, but it does provide a rule that the position vs time function must satisfy.

Solving the equation of motion

What we really want is $x = x(t)$, that satisfies (1). Finding this is called "solving the equation of motion." In general solving the equation of motion means:

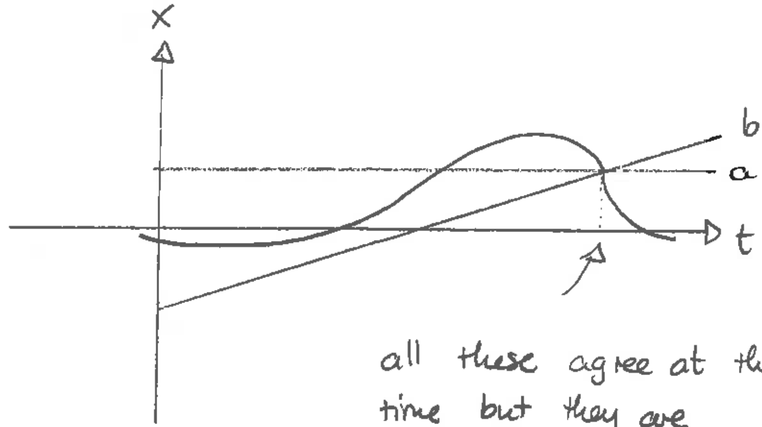
Find some function of time $x = x(t)$ so that at any time t, both sides of (1) are satisfied.

Quiz 1 $\frac{1}{7}$ → $4/7$ Quiz 2 $\frac{2}{7}$ → $7/7$

option 3 ambiguous [illegible] does work at time t.

Note that we do not seek a solution that works for one particular time, but rather a solution that works for all times. We can represent this situation graphically. The issue becomes:

"Which graph satisfies (1) at all times?"



all these agree at this time but they are different functions.

Note that graphically

$$\frac{d^2x}{dt^2} > 0 \Rightarrow \text{concave up}$$

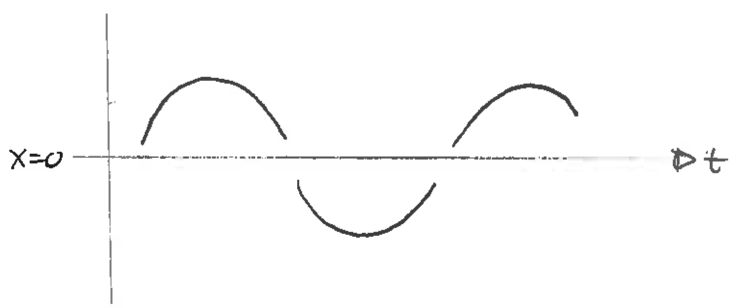
$$\frac{d^2x}{dt^2} < 0 \Rightarrow \text{concave down}$$

Noting that $k/m > 0$ we have:

If $x < 0$ then $\frac{d^2x}{dt^2} > 0 \Rightarrow$ concave up.

If $x > 0$ " $\frac{d^2x}{dt^2} < 0 \Rightarrow$ concave down.

So the graph must have a form of the illustrated type.



There are formal mathematical methods for solving Eq(1). But we can use graphical and physical insights to seek solutions. We <u>guess</u> the following:

$$x(t) \stackrel{??}{=} A\cos(\omega t).$$

where A and ω do not depend on t.

<u>Exercise</u>: Show that $x(t) = A\cos(\omega t)$ satisfies (1) provided that ω satisfies a special condition. Find the condition.

<u>Answer</u>: $$\frac{dx}{dt} = -\omega A \sin(\omega t)$$

$$\frac{d^2x}{dt^2} = -\omega A \omega \cos(\omega t) = -\omega^2 A\cos(\omega t)$$

$$= -\omega^2 x$$

It works if and only if $\omega^2 = k/m$ ▣

So we have:

$$\boxed{\begin{array}{l}\text{The equation of motion for the spring/mass system} \\ \quad \dfrac{d^2x}{dt^2} = -k/m\, x \\ \text{has as a possible solution:} \\ \quad x = A\cos(\omega t) \\ \text{where} \\ \quad \omega = \sqrt{k/m}\end{array}}$$

We can also see that for this solution:

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$$

which is the equation of motion for a <u>simple harmonic oscillator</u>. The units of ω are s^{-1}. To check

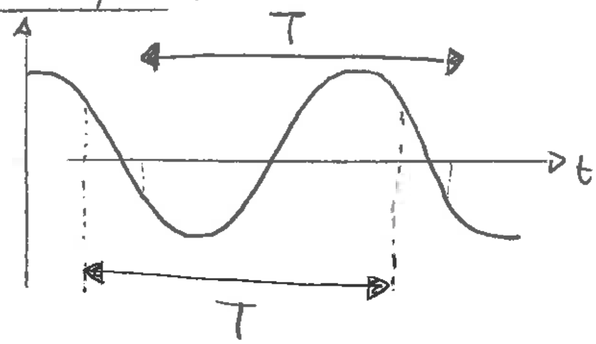
$$\left.\begin{array}{l} k \sim N/m = kg\, m/s^2/m = kg/s^2 \\ m \sim kg \end{array}\right\} \Rightarrow \omega \sim \sqrt{1/s^2} = s^{-1}$$

Properties of the solution

A graph of $x = A\cos(\omega t)$ indicates repeated motion

Slide 1

Note that this repeats at regular intervals. The period of oscillation is the shortest interval at which the pattern repeats.



This is denoted T. We can extract this mathematically by noting that

$$\cos(\omega t) = \cos(\omega t + n2\pi)$$

for any integer n. So

$$\cos(\omega t) = \cos\left(\omega\left(t + \frac{n2\pi}{\omega}\right)\right)$$

The pattern repeats at intervals spaced by times $\frac{n2\pi}{\omega}$ apart. The shortest occurs when $n=1$. So

$$\boxed{T = \frac{2\pi}{\omega}}$$

The frequency of oscillation is

$$\boxed{f = \frac{1}{T}}$$

and is the number of complete cycles of oscillation per second. This has units $s^{-1} \equiv$ Hertz (Hz). It follows that

$$\boxed{\omega = 2\pi f}$$

and this is called the angular frequency.

Exercise: Determine the time taken to complete N cycles in terms of frequency.

Answer: For one cycle time $= T$

N " s time $= NT = N/f \equiv \Delta t$

Thus $f = \frac{N}{\Delta t}$. If $\Delta t = 1s$ and N = number of cycles in 1 second, then $f = N$ cycles in one second.

The quantity A is called the amplitude of oscillation and is the maximum displacement from equilibrium.

Quiz 3

Here $V(t) = \frac{dx}{dt} = -\omega A \sin(\omega t)$.

and $a(t) = \frac{dv}{dt} = -\omega^2 A \cos(\omega t)$