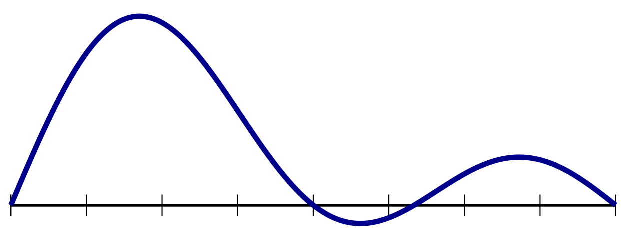
Phys 230
Fall 2013

Intermediate Dynamics: Group Exercises 11

Superpositions of Standing Waves

1 Waves on a string with fixed ends.

A string has fixed ends at $x = 0$ and $x = L$. In one particular situation, a snapshot of the string at $t = 0\,\text{s}$ is illustrated.



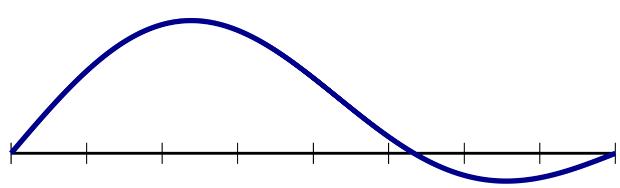
a) Does this satisfy the boundary conditions for a string with fixed ends? Does it resemble any of the known standing waves for a string with both ends fixed?

b) One possibility for describing such situations is via linear combinations, or superpositions, of standing waves. Consider

$$y(x,t) = A_1 \sin(k_1 x) \cos \omega_1 t + A_2 \sin(k_2 x) \cos \omega_2 t$$

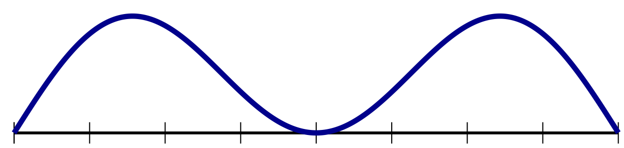
where $k_n = n\pi/L$ and $\omega_n = k_n v$. Show that this satisfies the wave equation and the boundary conditions for a string with both ends fixed, regardless of the values of A_1 and A_2. Consider the displacement of the string at $x = L/4$. Does this oscillate with one frequency or not?

c) Consider the string whose initial displacement is as illustrated.



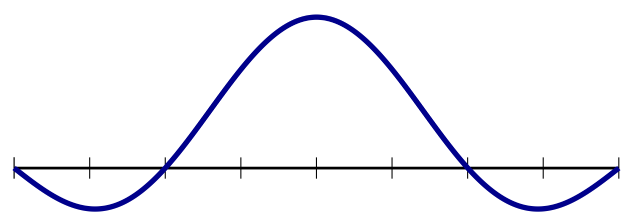
This could possibly be represented by superposition of standing waves. Suggest which standing waves for a string with both ends fixed might be able to produce the illustrated pattern.

d) Consider the string whose initial displacement is as illustrated.



This could possibly be represented by superposition of standing waves. Suggest which standing waves for a string with both ends fixed might be able to produce the illustrated pattern.

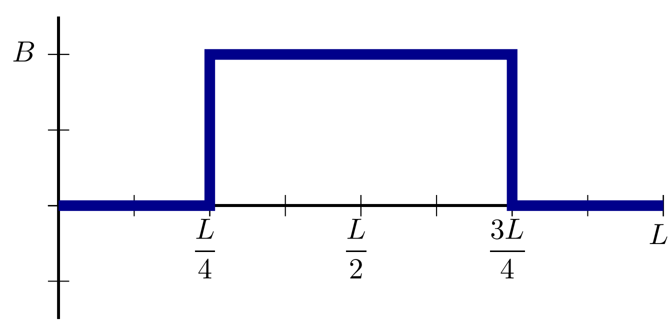
e) Consider the string whose initial displacement is as illustrated.



This could possibly be represented by superposition of standing waves. Suggest which standing waves for a string with both ends fixed might be able to produce the illustrated pattern.

2 Fourier Analysis

A string has fixed ends at $x = 0$ and $x = L$. The displacement along the string at $t = 0\,\text{s}$ is illustrated below



Determine the Fourier coefficients for this.