Intermediate Dynamics: Group Exercises 9

Traveling Waves

1 Sinusoidal traveling waves

Consider the sinusoidal traveling wave on a string described by

$$y(x,t) = A\sin\left[k(x-vt)\right].$$

Suppose that a snapshot of this is taken at $t = 0$.

a) Determine an expression for the profile of the string at $t = 0$.

b) Determine values of x at which the first three maxima to the right of $x = 0$ occur. Use this to determine an expression for the wavenumber k in terms of the wavelength.

Consider the sinusoidal traveling wave on a string described by

$$y(x,t) = A\sin\left[k(x+vt)\right].$$

c) Suppose that the point $x = 0$ on the string is observed. Determine an expression for the displacement of this point as a function of time. Show that this predicts that the point oscillates. Determine an expression for the angular frequency of oscillation in terms of k and v.

d) Show that the frequency of oscillation, f, and wavelength, λ are related by

$$\lambda f = v.$$

2 Sinusoidal traveling waves and the wave equation

Show by direct substitution that

$$y(x,t) = A\cos\left(kx - \omega t\right)$$

satisfies the wave equation provided that k and ω satisfy a particular relationship. Determine this relationship.

3 General wave equation solutions

The wave equation is

$$\frac{\partial^2 y}{\partial t^2} = v^2\,\frac{\partial^2 y}{\partial x^2}.$$

a) Show that $y(x,t) = A\,(x - vt)^n$ where A is a constant and n is an integer satisfies the wave equation.

b) Show that $y(x,t) = A\,(x + vt)^n$ where A is a constant and n is an integer satisfies the wave equation.

Neither of these represents a real physical solution to the wave equation but each illustrates a mathematical solution to the equation.