Phys 230
Fall 2013

Intermediate Dynamics: Group Exercises 7

Driven Damped Oscillator

1 Solutions to the equation of motion

The equation of motion for a damped driven oscillator is

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t \tag{1}$$

Consider the following candidate for the steady state solution to the equation of motion:

$$x(t) = A\cos(\omega t - \delta).$$

a) A preliminary step involves rewriting the right hand side of Eq. (1). Note that $\cos(\omega t) = \cos(\omega t - \delta + \delta)$. Use this and a trigonometric identity to rewrite $\cos(\omega t)$ as a linear combination of $\cos(\omega t - \delta)$ and $\sin(\omega t - \delta)$.

b) Substitute $x(t)$ into the rewritten version of Eq. (1) and express the result as a linear combination of $\cos(\omega t - \delta)$ and $\sin(\omega t - \delta)$. Use this to determine conditions that A and δ must satisfy. Solve these to get expressions for A and δ.

2 Phase of oscillation for a driven damped oscillator.

The phase of a driven damped oscillator is.

$$\delta = \arctan\left[\frac{\omega\gamma}{\omega_0^2 - \omega^2}\right].$$

The aim of this exercise is to explore this phase as a function of driving frequency and relate it to the motion of the oscillator.

a) Rewrite δ in terms of Q and $u = \omega/\omega_0$.

b) Suppose that $\omega \to 0$. Determine an approximate expression for δ. What does your result imply for the motion of the oscillator relative to the driving force?

c) Suppose that $\omega \to \infty$. Determine an approximate expression for δ. What does your result imply for the motion of the oscillator relative to the driving force?

d) Suppose that $\omega = \omega_0$. Determine an approximate expression for δ. What does your result imply for the motion of the oscillator relative to the driving force?

e) Sketch a plot of δ as a function of u.