Intermediate Dynamics: Group Exercises 6

Damped Harmonic Oscillator

1 Damped harmonic oscillator

The equation of motion for a damped harmonic oscillator is

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0. \tag{1}$$

We shall aim to find damped oscillations described by

$$x(t) = Ce^{-\alpha t} \cos(\omega t)$$

where $\alpha > 0$ and $\omega > 0$ are constants *(note that the symbols ω and ω_0 are intentionally different)*.

a) Assuming this type of solution determine an expression for $\frac{dx}{dt}$.

b) Determine an expression for $\frac{d^2x}{dt^2}$.

c) Substitute these into Eq. (1) and gather all terms with $\cos(\omega t)$ and all with $\sin(\omega t)$. Determine conditions on α and ω such that the resulting equation is satisfied at all times.

d) Is this solution valid for all values of γ and ω_0?

2 Lightly damped oscillations

The amplitude for a lightly damped oscillator is

$$C(t) = C_0 e^{-\gamma t/2}$$

where C_0 is a constant.

a) Determine $C(1\,\mathrm{s})/C(0\,\mathrm{s})$.

b) Determine $C(2\,\mathrm{s})/C(1\,\mathrm{s})$.

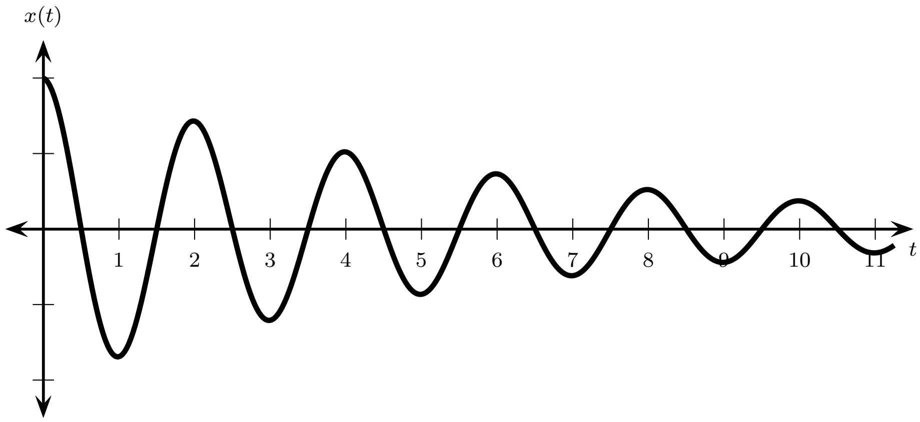
c) Determine $C(t + 1\,\mathrm{s})/C(t)$ for any time t.

d) Determine γ from the ratio $C(t + 1\,\mathrm{s})/C(t)$ for any time t.

e) Determine γ from the ratio $C(t + T)/C(t)$ where T for any time t but where T is the period of oscillation.

3 Determining the damping constant

A graph of position vs. time for a lightly damped oscillator is shown. The time is measured in seconds.



Determine γ using the data from the graph.