Intermediate Dynamics: Group Exercises 3

Spacetime and Momentum

1 Supernovae

Two stars α and β are at rest with respect to a space station. Star α is 30 lt yr to the left of the space station and star β is 30 lt yr to the right of the space station. Both stars undergo supernova explosions and according to the space station observer, the resulting light flashes from α and β arrive simultaneously at $t = 30\,\text{yr}$. A rocket travels to the right with constant velocity $\frac{4}{5}c$ and it passes the space station when $t = 0\,\text{yr}$.

a) Determine the times at which each supernova occurred according to the space station observers. Denote these by t_α and t_β. Determine $\Delta t = t_\beta - t_\alpha$.

b) Determine the times at which each supernova occurred according to the rocket observers. Denote these by t'_α and t'_β. Determine $\Delta t' = t'_\beta - t'_\alpha$. Which supernova occurred first according to the rocket observers?

c) Sketch superimposed spacetime diagram axes for the space station (unprimed) and the rocket (primed). Sketch the worldlines of the space station, the rocket ship and the two stars prior to the supernova.

d) The space station emits a test flash of light at $t = 0\,\text{yr}$ in the direction of star β. Sketch the worldline of this light pulse on the spacetime diagram.

e) The space station emits a test flash of light at $t = 10\,\text{yr}$ in the direction of star α. Sketch the worldline of this light pulse on the spacetime diagram. What is the slope of the worldline of a light pulse?

f) Sketch the worldline of light flash from supernova α and do the same for β.

g) Use the space time diagram to determine which supernova occurs first according to the rocket observers.

h) Consider the two events: i) supernova α occurs and ii) supernova β occurs. Determine the position coordinates of these events x_α and x_β according to the space station observers. Determine $\Delta x = x_\beta - x_\alpha$ and also $(\Delta s)^2 := -(c\Delta t)^2 + (\Delta x)^2$.

i) Repeat the previous part for the rocket observers and determine $(\Delta s')^2 := -(c\Delta t')^2 + (\Delta x')^2$. Is $(\Delta s')^2 = (\Delta s)^2$?

2 Classical momentum conservation?

In classical physics, a key law is that for an isolated system the total momentum is conserved, where the momentum of a particle with mass m and velocity $\mathbf{v}$ is $\mathbf{p} = m\mathbf{v}$. The momentum of a system of particles can be computed by observing the motion of the particles in one inertial frame, computing individual momenta, and adding to get the total. Suppose that, according to this frame, the total momentum is conserved. Will it be conserved according to any other inertial frame. A key finding is that if the total momentum as determined in one inertial frame is conserved, then for the same situation, viewed from a different inertial frame related by Galilean transformations, teh total momentum is also conserved? Will this be true when frames are related by Lorentz transformations?

Consider particle A, with mass m, initially moving with velocity $\mathbf{v}_i = v_0\hat{\boldsymbol{\imath}}$ as viewed in the frame in which the lab is at rest. It collides with particle B, with mass $2m$, initially at rest. Particle A moves left with velocity $-v_0$ and B right with velocity v_0.

a) Show that, in the lab frame, classical momentum is conserved.

Now view the situation from a frame that moves with velocity $\mathbf{u} = u\hat{\boldsymbol{\imath}}$ with respect to the lab frame. *As a first pass you could specialize to* $u = v_0$.

b) Using the results from the previous part and the relativistic velocity transformations, determine the velocities of the two particles before and after the collision.
c) Determine the total momentum before the collision.
d) Determine the total momentum after the collision.
e) Is the total momentum conserved in this frame?