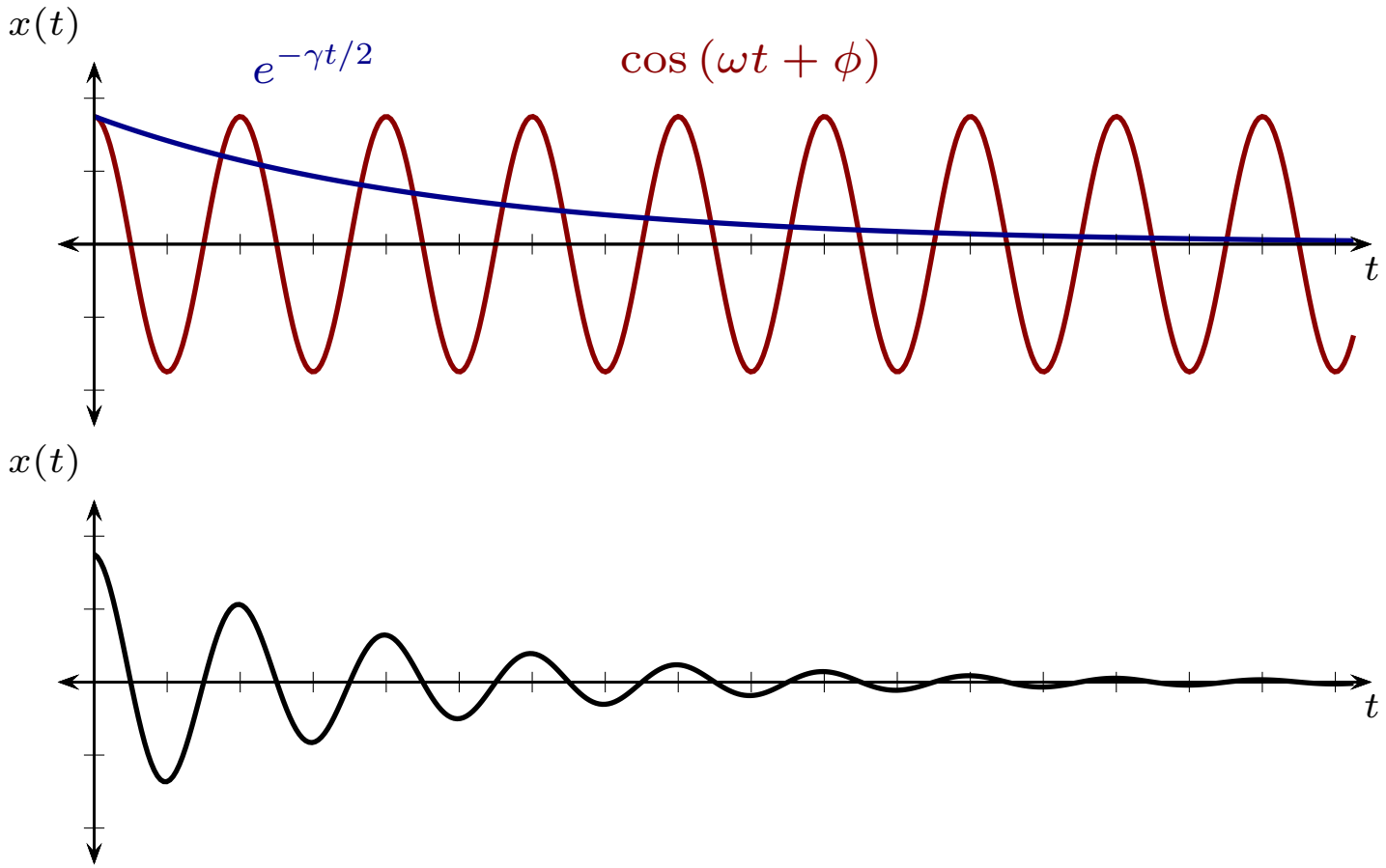
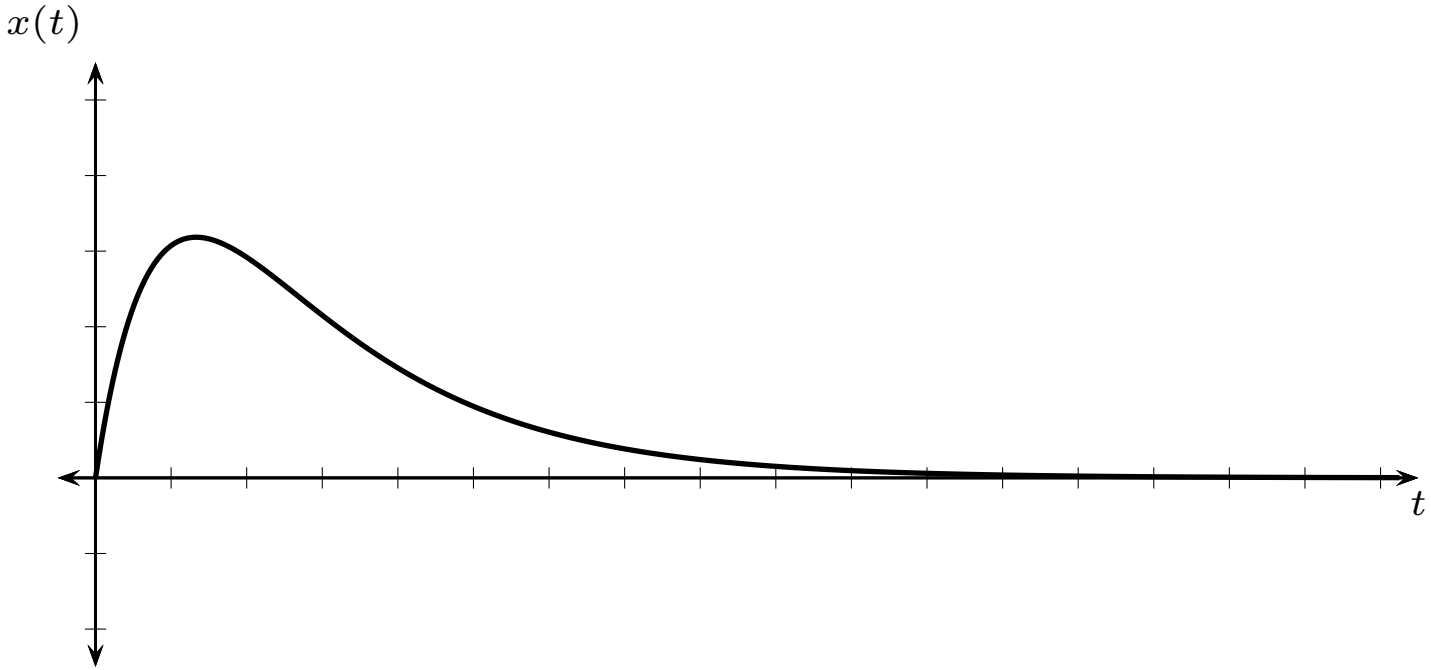
Damped Harmonic Motion

For a solution of the form $x(t) = Ce^{-\gamma t/2} \cos(\omega t + \phi)$:



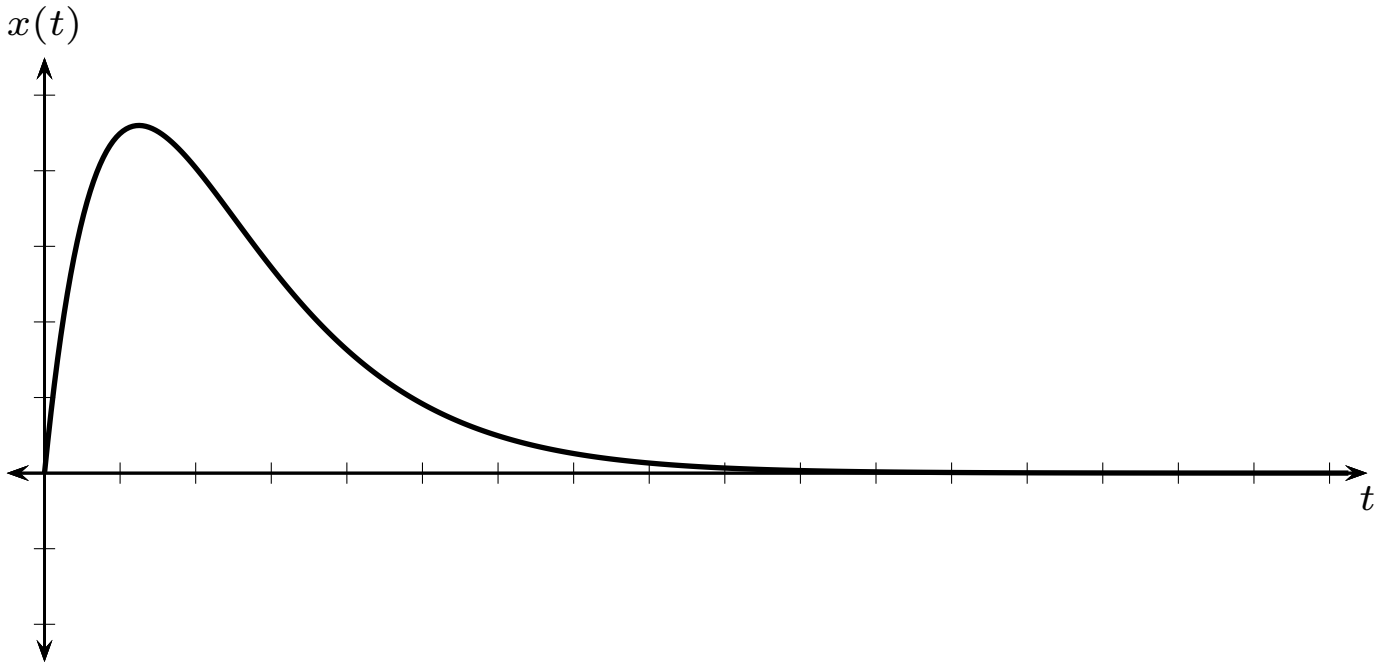
Overdamped Motion

For a solution of the form $x(t) = e^{-\gamma t/2}\left\{Ae^{\alpha t} + Be^{-\alpha t}\right\}$ with $x(0) = 0$.



Critically damped Motion

For a solution of the form $x(t) = (A + Bt)\, e^{-\gamma t/2}$ with $x(0) = 0$.



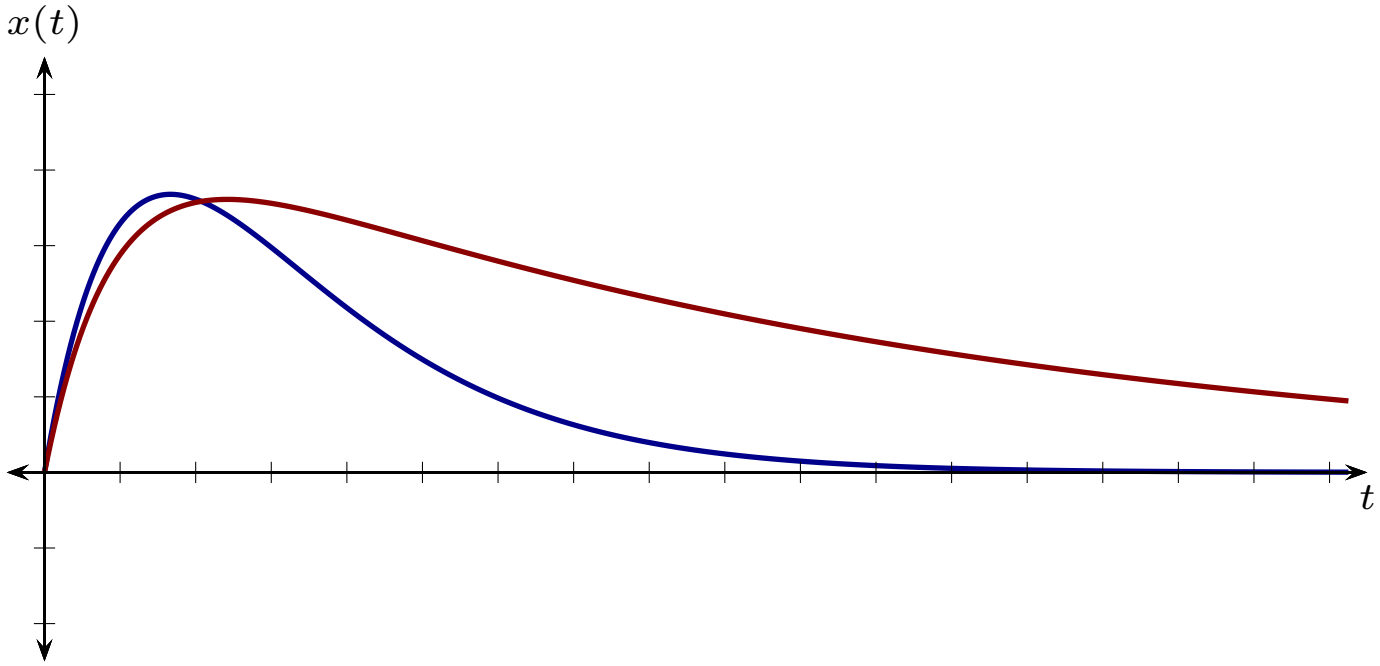
Critically damped Motion

For critically damped motion (graph blue) with $x(0) = 0$, position is

$$x(t) = v_0 t \; e^{-\gamma t/2}.$$

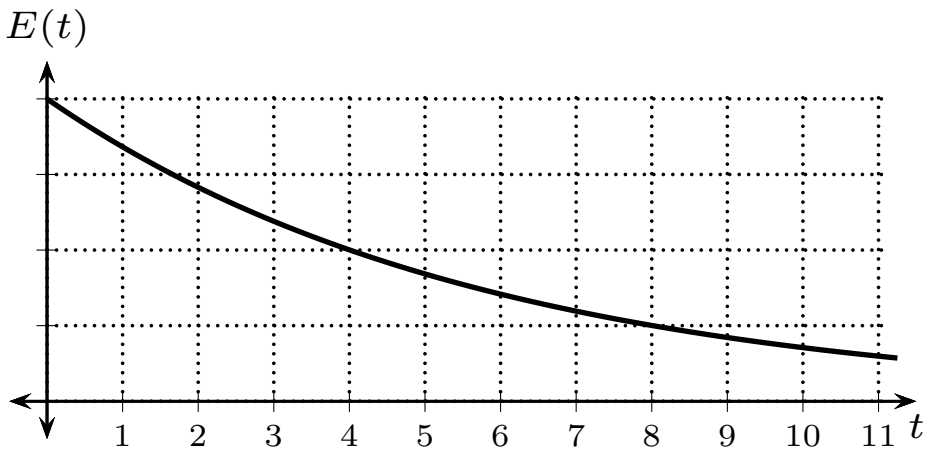
For heavily damped motion (graph red) with $x(0) = 0$, position is

$$x(t) = \frac{v_0}{2\alpha} \; e^{-\gamma t/2} \left\{ e^{\alpha t} - e^{-\alpha t} \right\}.$$



Question 1

The following is a graph of energy vs. time for a damped oscillator. Time is measured in units of seconds.

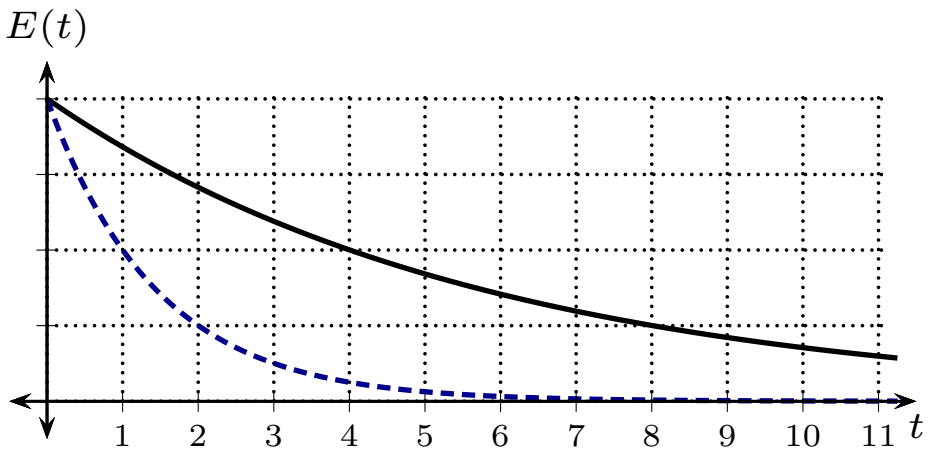


Which of the following is true?

1. $\gamma = \frac{1}{2} \ln 2$
2. $\gamma = \frac{1}{4} \ln \left(\frac{1}{2}\right)$
3. $\gamma = -\frac{1}{4} \ln \left(\frac{1}{2}\right)$
4. $\gamma = 4 \ln \left(\frac{1}{2}\right)$

Question 2

The following are graphs of energy vs. time for damped oscillators. Time is measured in units of seconds.

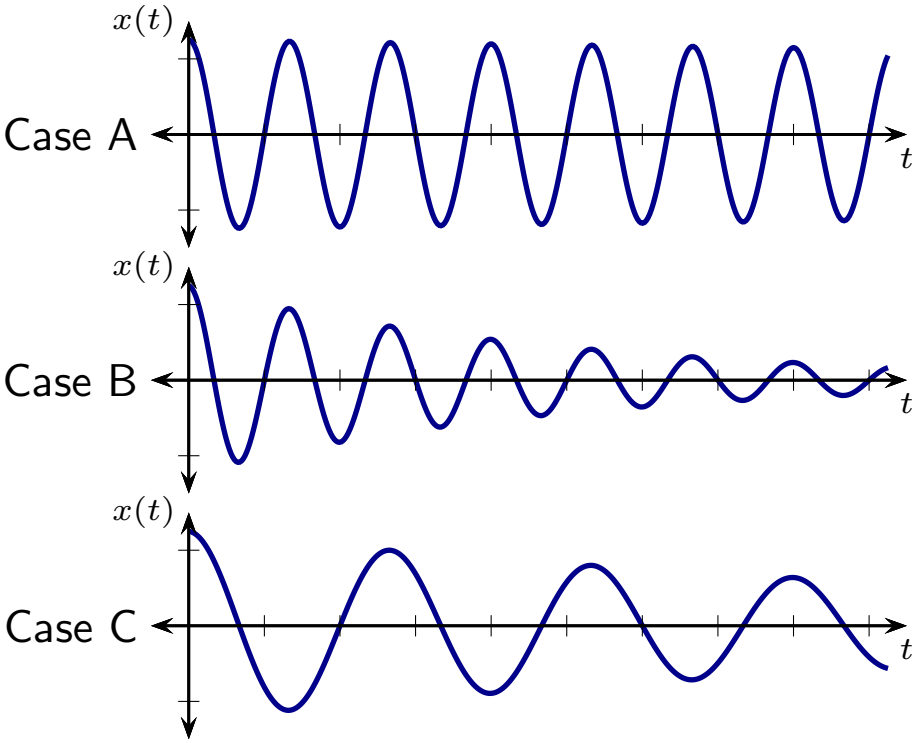


Denote the oscillator for which the curve is solid by A and the other oscillator by B. Which of the following is true?

1. $\gamma_A = 4\gamma_B$
2. $\gamma_A = 2\gamma_B$
3. $\gamma_A = \gamma_B$
4. $\gamma_A = \frac{1}{2}\,\gamma_B$
5. $\gamma_A = \frac{1}{4}\,\gamma_B$

Question 3

Three mass/spring oscillators produce:



Which of the following is true?

1. $Q_{\mathrm{A}} = Q_{\mathrm{B}} = Q_{\mathrm{C}}$
2. $Q_{\mathrm{A}} > Q_{\mathrm{B}} = Q_{\mathrm{C}}$
3. $Q_{\mathrm{A}} < Q_{\mathrm{B}} = Q_{\mathrm{C}}$
4. $Q_{\mathrm{A}} > Q_{\mathrm{B}} > Q_{\mathrm{C}}$
5. $Q_{\mathrm{A}} < Q_{\mathrm{B}} < Q_{\mathrm{C}}$