Intermediate Dynamics: Homework 20

Due: 6 December 2013

1 King, *Vibrations and Waves*, 7.5, page 181.

2 King, *Vibrations and Waves*, 7.6, page 181.

3 King, *Vibrations and Waves*, 7.11, page 182.

4 Interference from coherent sources

Two sources of waves, S_1 and S_2, located a distance d apart, produce circular waves with the same amplitudes, wavenumbers and frequencies but different phases. Their configuration is illustrated in Fig. 1. Denote the wavenumber by k and the angular frequency by ω.

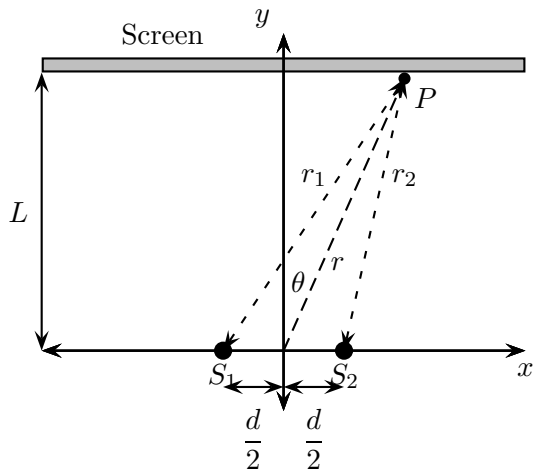

Figure 1: Question 4. Sources emitting circular waves. Screen location relative to sources.

The general form of a wave in this medium is given by $y(x,t) = C\cos(kr - \omega t + \phi_1)$ and for such a wave the intensity is γC^2 where γ is a constant. The displacements produced by the two waves at a point P are

$$y_1(P,t) = A\cos(kr_1 - \omega t + \phi_1)$$
$$y_2(P,t) = A\cos(kr_2 - \omega t + \phi_2)$$

where r_1 is the distance from source 1 to P and r_2 is the distance from source 2 to P.

a) Determine an expression for $y(P,t) = y_1(P,t) + y_2(P,t)$ and use the result to show that the intensity at P is

$$I = 4\gamma A^2 \cos^2\left(\frac{k(r_1 - r_2)}{2} + \frac{(\phi_1 - \phi_2)}{2}\right).$$

The intensity distribution clearly depends on the phase difference $\Delta\phi = \phi_1 - \phi_2$. Provided that the phase difference remains fixed, and regardless of its actual value, the two sources are said to be *coherent.*

b) Now suppose that $r \gg d$. Then use $r_1 - r_2 \approx d\sin\theta$ find an expression, in terms of the wavelength λ, source separation d and phase difference $\Delta\phi$, for the angles at which intensity maxima and minima occur.

c) The locations of intensity maxima and minima (fringes) clearly depend on $\Delta\phi$. Suppose that the sources which are *in phase*, i.e. $\Delta\phi = 0$. Find expressions for the angles at which maxima an minima occur.

d) Suppose that the sources which are *out of phase*, i.e. $\Delta\phi = \pi$. Find expressions for the angles at which maxima an minima occur. How do these angles compare to the case for sources that are in phase?

The visibility of the fringes is defined to be

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

where $I_{\max}$ and $I_{\min}$ are the maximum and minimum intensities as one scans across the screen.

e) Determine the visibility for both the in phase and out of phase cases.

f) Suppose that the phase difference rapidly and randomly fluctuates between all possible values. What visibility would result?