# Intermediate Dynamics: Homework 15

Due: 13 November 2013

## 1 Complex functions and the damped oscillator

The equation of motion for a damped oscillator is

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0. \qquad (1)$$

a) Consider

$$\frac{d^2z}{dt^2} + \gamma \frac{dz}{dt} + \omega_0^2 z = 0. \qquad (2)$$

Assume that the solution to this has form

$$z(t) = De^{ut}$$

where $D$ and $u$ are complex constants. Substitute this into Eq. (2) and find an algebraic expression for $u$. Does this solution work for all possible values of $D$?

b) Show that the real and imaginary parts of $u$ will depend on whether $\gamma > 2\omega_0$ or $\gamma < 2\omega_0$.

c) Use your expression for $z(t)$ to determine the solution to Eq. (1) if $\gamma > 2\omega_0$.

d) Use your expression for $z(t)$ to determine the solutions to Eq. (1) if $\gamma < 2\omega_0$.

## 2 Complex representation for a simple harmonic oscillator

A spring and mass system oscillates with angular frequency $20\,\text{rad/s}$. At $t = 0\,\text{s}$, the displacement of the mass is $0.40\,\text{m}$ and its velocity is $-8.0\,\text{m/s}$.

a) The general form for the position of the oscillator is

$$x(t) = C\cos(\omega t + \phi)$$

Determine expressions for $C$ and $\phi$.

b) Determine an expression for the complex representation for this particular oscillator, i.e.

$$z(t) = De^{i\omega t}.$$

Determine the complex constant $D$ that represents this motion.

c) One way to partly check your solution is to use the fact that the particle velocity satisfies

$$v = \text{Re}\left[\frac{dz}{dt}\right].$$

Use this to determine the initial velocity. Does it matches that given above?