Phys 230
Fall 2013

Intermediate Dynamics: Homework 14

Due: 11 November 2013

1 Algebraic operations on complex numbers

a) Express $(3+6i)+(5-3i)$ in the form $z = a + ib$ for real a and b.

b) Express $(3+6i)(5-3i)$ in the form $z = a + ib$ for real a and b.

c) Express $\dfrac{3+6i}{5-3i}$ in the form $z = a + ib$ for real a and b.

d) Find $z = a + ib$ where a and b are real such that $(3+6i)+z = 0$.

e) Demonstrate explicitly that $(3+6i)^*(5-3i)^* = \left[(3+6i)(5-3i)\right]^*$. This is an example of the general rule that

$$z_1^* z_2^* = (z_1 z_2)^* \tag{1}$$

2 The complex plane

a) Represent the following complex numbers in the complex plane. Use one set of axes for all of them.

$$z_1 = 2 \tag{2}$$
$$z_2 = -2i \tag{3}$$
$$z_3 = 1 + 2i \tag{4}$$
$$z_4 = 1 - 2i \tag{5}$$
$$z_5 = -1 + 2i \tag{6}$$
$$z_6 = -1 - 2i \tag{7}$$

b) Suppose that z satisfies $|z| = 1$. Indicate the set of all possible locations of this number in the complex plane. Pick any one of these points in your diagram which lies in the first quadrant. Indicate the angle θ corresponding to this point on your diagram and use trigonometric reasoning to show that

$$z = \cos\theta + i\sin\theta.$$

3 Complex exponential functions

For each of the following determine the simplest possible expression for $\text{Re}[z]$.

a) $z = 2e^{i(\omega t + \pi/2)}$.

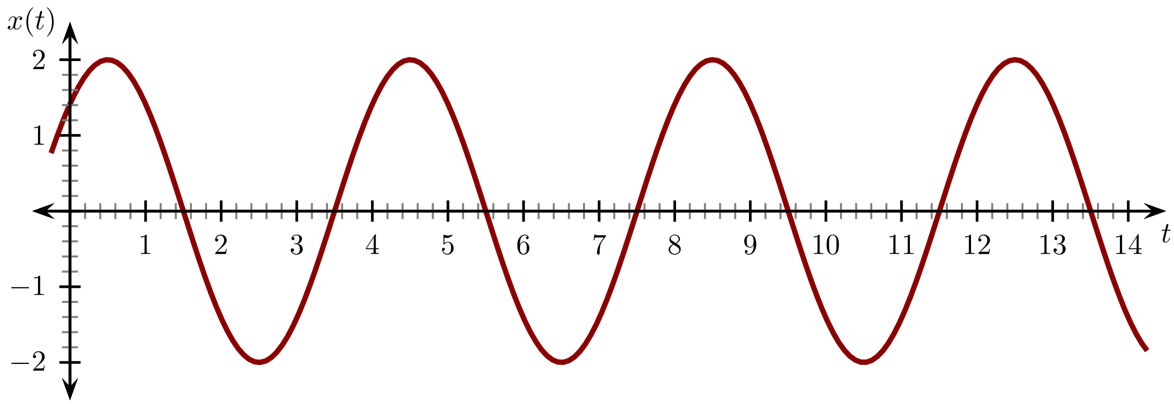
b) $z = Ae^{i\omega t}$ where $A = 3e^{i\pi/4}$.

c) $z = Ae^{i\omega t}$ where $A = (1-i)/\sqrt{2}$.

d) $z = 2e^{i(\omega t + i\gamma t)}$.

4 Representations of Oscillatory Motion

A block attached to a spring oscillates in such a way that its position is represented by the following graph where the horizontal axis is in units of seconds and the vertical in units of meters.



a) The general solution to the equation of motion for the simple harmonic oscillator has the form $x(t) = C\cos(\omega t + \phi)$ where C, ω and ϕ are constants. Determine explicit values of C and ω for the motion graphed above.

b) Determine a value of t for which $\cos(\omega t + \phi) = 1$ and use the result to determine an explicit value for ϕ.

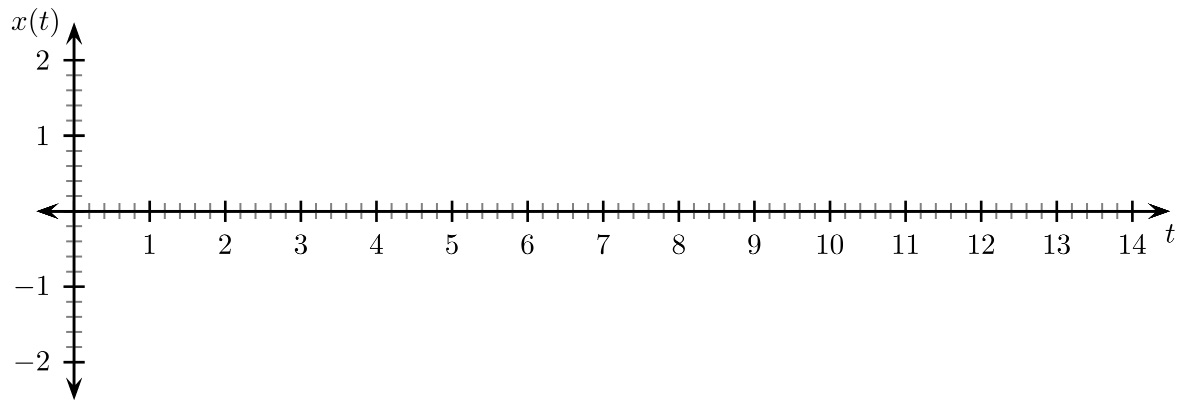
c) Express the position in the form $x(t) = A\cos(\omega t) + B\sin(\omega t)$ by providing explicit values for A and B. Use this to determine the initial position and velocity of the oscillator.

d) Find the complex representation of the position, $z(t)$, such that $x(t) = \mathrm{Re}[z(t)]$. Write this in the form $Z(t) = De^{i\omega t}$ by providing ω and the complex number D.

e) Sketch on the axis below the expression for $x(t)$ given that the complex representation for this is

$$z(t) = De^{i\pi t}$$

where $D = \frac{1}{\sqrt{2}}(-1+i)$.



5 King, *Vibrations and Waves*, 3.8, page 75.

Complete the problem by following these steps.

a) Sketch the pendulum at rest and also when its support is displaced. Label the *horizontal displacement* of the pendulum from equilibrium by x. Label the displacement of the support by ξ. Ignore the vertical displacement.

b) Applying Newton's second law (not its rotational form) to the pendulum to obtain the equation of motion. This will involve geometry and trigonometry with the variables x and ξ and you will have to use the approximation that the length of the pendulum satisfies $l \gg |x - \xi|$ to obtain the standard equation of motion for a driven damped oscillator.

c) Recast the equation of motion into complex form. This completes part a) of the text problem.

d) Now complete part b) of the text problem.