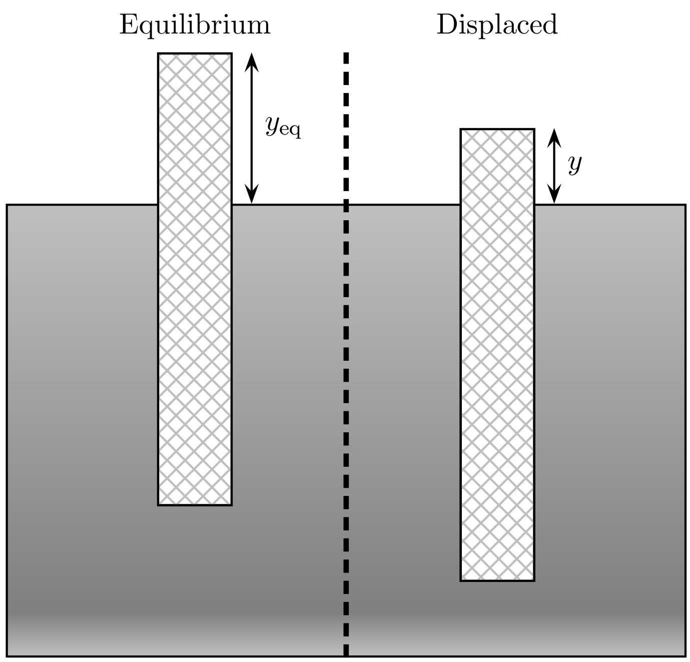
Phys 230
Fall 2013

Intermediate Dynamics: Homework 10

Due: 28 October 2013

1 Oscillating floating object

A metal cylinder of mass m, length L and cross sectional area A is suspended end up in a liquid of density ρ as illustrated below.



a) By considering all the forces acting on the cylinder and assuming that the air pressure at the top of the cylinder is the same as that at the surface of the liquid, show that the equilibrium position of the top of the cylinder is

$$y_{\text{eq}} = L - \frac{m}{\rho A} \tag{1}$$

b) Now suppose that the cylinder is displaced. By applying Newton's second law and using Eq. (1), show that

$$m\frac{d^2 y}{dt^2} = \rho g A(y_{\text{eq}} - y).$$

c) The displacement from equilibrium is $y_d := y - y_{\text{eq}}$. Show that

$$m\frac{d^2 y_d}{dt^2} = -\rho g A y_d.$$

Use this to determine an expression for the angular frequency of oscillation of the cylinder.

d) Suppose that a log of mass 500 kg and diameter 0.50 m floats in this fashion in water. Determine the period of oscillation of the log if it manages to stay oriented end up.

e) Describe how you could use the oscillations of a cylinder of known mass and cross-sectional area to determine the density of an unknown liquid.

2 LC circuits

The PASCO RLC circuit board has an inductor for which $L = 8.2 \times 10^{-3}\,\mathrm{H}$ and a capacitor for which $C = 100 \times 10^{-6}\,\mathrm{F}$.

a) Determine the angular frequency and period of the oscillating current.

b) Suppose that the total energy of the oscillating system is 1.0 J. Determine the maximum charge on the capacitor.

3 Damped harmonic oscillator

A damped harmonic oscillator satisfies the following equation of motion

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0.$$

The solution for underdamped motion is

$$x = Ce^{-\gamma t/2}\,\cos(\omega t + \phi),$$

where $\omega = \sqrt{\omega_0^2 - \gamma^2/4}$. Suppose that the oscillator is initially at rest at position x_0. Determine values of or expressions for C and ϕ in terms of x_0.

4 Heavily damped oscillator

An oscillator is heavily damped if $\gamma > 2\omega_0$.

a) Show that the solution to the equation of motion can be expressed as

$$x(t) = Ae^{-\gamma t/2}e^{\kappa t} + Be^{-\gamma t/2}e^{-\kappa t}$$

and find an expression for κ in terms of m, γ, ω_0. Verify that κ is real.

b) Determine an expression for the velocity of the oscillator.

c) Suppose that the oscillator is initially at $x = 0$ and at this moment moves with speed v_0. Determine expressions for A and B in terms of v_0 and κ. You should be able to show that

$$x(t) = \mathrm{const} \times e^{-\gamma t/2}\left(e^{\kappa t} - e^{-\kappa t}\right) = \mathrm{const} \times e^{-\gamma t/2}\sinh(\kappa t)$$

d) If the damping is extremely heavy then $\gamma \gg 2\omega_0$. Find an approximate expression for κ in this case and use this to find an approximate expression for $x(t)$ given that the oscillator is initially at $x = 0$ and at this moment moves with speed v_0. In this situation, determine an expression for the maximum force exerted on the oscillator.