Intermediate Dynamics: Homework 9

Due: 23 October 2013

1 Oscillating spring and mass

A block of mass 5.0 kg is attached to a spring with spring constant 500 N/m. This is initially compressed by 0.20 m and given a push, compressing it further, with speed 2.5 m/s.

a) The general solution expression for the position of the mass is

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

Determine A, B and ω for this system.

b) An alternative general solution expression for the position of the mass is

$$x(t) = C\cos(\omega t + \phi)$$

Determine C and ϕ for this system.

c) Determine the initial kinetic and potential energy for this system. Use these to determine the total energy.

d) Verify that $E = \frac{1}{2}\, kC^2$.

2 King, *Vibrations and Waves*, 1.6, page 30. To do this:

a) First answer part b). You have to account for the gravitational force. To deal with this consider three situations: the unstretched spring without any suspended mass, the spring and mass in equilibrium and stretched spring with mass that is moving. Define the vertical position variable, y, as the displacement from equilibrium.

b) Determine an expression for the position of the unstretched spring in terms of k, m and g. This will enter into subsequent parts of the problem.

c) Use Newton's second law to determine the net force on the mass when it is displaced away from equilibrium. Use this to determine an expression for the equation of motion for the mass. Do you need to solve this to identify the frequency of oscillation? Identify the frequency of oscillation.

d) Repeat this by starting by finding the total energy of the system and using the resulting expression to identify the frequency of oscillation. Note that a more general version of the expression for the energy of a simple harmonic oscillator is

$$E = \alpha\left[\left(\frac{dx}{dt}\right)^2 + \omega^2 x^2\right] + E_0$$

where α and E_0 are constants.

e) Convince yourself that for part b), the Earth's gravitational force is irrelevant. You can therefore ignore it in parts a) and c). Now perform both the Newton's second law analysis and the energy analysis to answer parts a) and c).